# Chaotic oscillator based on fractional order memcapacitor ☆

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**Abstract**

Many literatures have discussed fractional order memristor and memcapacitor based chaotic oscillators but the entire oscillator model is considered to be fractional order. My interest is to propose a nonlinear oscillator with only the memcapacitor element considered fractional order. Hence I propose a fractional order memcapacitor based novel chaotic oscillator. The complete mathematical model for the proposed oscillator is derived and presented. The dimensionless state equations are then analysed using the equilibrium points and their stability, Eigen values, Kaplan-Yorke dimensions and Lyapunov exponents. To understand the complete dynamical behaviour, bifurcation graphs are obtained and presented. Finally the proposed fractional memcapacitor oscillator is implemented using off the shelf components.

*Keywords:*
**Nonlinear systems, chaos, memcapacitor, fractional order circuit design.**

## 1. Introduction

Chaos theory has become very popular in recent years and many studies have been carried out on chaotic systems. Researchers have been exploring different chaotic systems [1, 2, 3], especially hidden [4, 5] and multi-stability attractors [6, 7]. Also, in the recent years, fractional order chaotic systems have become very popular and interesting topic. Many researchers have worked on them [8, 9, 10].

Chaos based applications are an important subject in the both science and engineering fields. Chaotic systems have used in fields such as oscillator [11, 12], random number generator [13], cryptology [14], steganography [15], synchronization [16], control [17], communication [18] and parameter estimation [19]. With improvements in circuit design technology through the development of integrated circuits [20], chaotic systems have aroused special interest, especially fractional order based chaotic systems[21].

*January 4, 2019*

Designing nonlinear oscillators with memristor elements have gained interest in the recent years. Recently there have been many literatures discussing about such oscillators. Recently many researchers have worked on the fractional order memristor (fracmemristor) models [22, 23, 24, 25, 26, 27]. There have been discussions on using memristor, memcapacitor, cubic nonlinear resistors, piecewise linear functions being used as the nonlinear element to generate chaos and hyperchaos [28, 29, 30, 31, 32, 33, 34]. Some special phenomena such as hidden attractors as well as coexisting attractors in memristor based oscillators are also discussed in the literatures [32, 33, 34]. Fractional order memristor based nonlinear oscillators have also been proposed and investigated [25, 35, 36] where the authors derived fractional order from the integer order model.

Fractional calculus has history dating back to 17th century but it found its applications in science and engineering research only in the recent years [37, 38, 39, 40, 41, 42]. Many physical systems such as dielectric polarization, electromagnetic waves, and quantum evolution of complex systems exhibit fractional order dynamics and thus fractional order control algorithms are achieving the attention of researchers [43, 44, 45, 46].

In this study, a chaotic oscillator based on fractional order memcapacitor is investigated. Firstly, a fractional order chaotic system is derived from fractional order memcapacitor circuit given in Fig. 1. Then, electronic circuit realization of the derived fractional order chaotic system is implemented. While it is not possible to implement fractional order integration with ordinary circuit elements, it was implemented with ordinary circuit elements using the approximated transfer function of the fractional order described in [46].

The article is organized as: in Section 2, chaotic oscillator with fractional order memcapacitor is introduced; in Section 3, dynamic analysis of the fractional order memcapacitor based chaotic (FMC) system is performed; in Section 4, circuit realization of FMC system is implemented and in the last section, Section 5, the conclusion is given.

## 2. Chaotic oscillator with fractional order memcapacitor

Developing fractional order nonlinear elements have gained interest in the recent years. Recently many researchers have worked on the fractional order memristor (fracmemristor) models [22, 23, 24, 25, 26, 27].

The Ohmic relationship of a memristor is given by

$$R_{in} = \frac{L_{doped}}{L_{total}} R_{on} + (1 - \frac{L_{doped}}{L_{total}}) R_{off} \quad (1)$$

where $R_{on}$ is the minimum resistance and $R_{off}$ maximum resistance of the memristor. The rate of change of $\frac{L_{doped}}{L_{total}}$ is given as,

$$\frac{d(\frac{L_{doped}}{L_{total}})}{dt} = \pm\frac{\mu_m R_{on}}{D^2}\, i(t)G(\frac{L_{doped}}{L_{total}}) \tag{2}$$

where $\mu_m$ denoting the dopant mobility, the length of memristor and $g(\frac{L_{doped}}{L_{total}})$ is dopant drift given by $f(\frac{L_{doped}}{L_{total}}) = 1 - (2\frac{L_{doped}}{L_{total}} - 1)^{2p}$ . The fractional memristor model is given by the relation

$$D^q x = \pm\frac{\mu_m R_{on}}{D^2}\, i(t)g(\frac{L_{doped}}{L_{total}}) \tag{3}$$

Solving (3) with (1), the input resistance of the memristor is derived as,

$$\frac{d^q R_{in}}{dt^q} = \pm\frac{\mu_m R_{on}}{D^2}\, i(t)g(\frac{L_{doped}}{L_{total}}) \tag{4}$$

where $R_d = R_{off} - R_{on}$. For linear window $g(\frac{L_{doped}}{L_{total}})$= 1 and using Riemann - Lioville Theorem the memristor resistance can be derived as

$$R_{in} = \left( R_{in}^{q+1} \mp q\,(q+1)\,kR_d \int\limits_0^t (t-\tau)^{q-1} v\,(\tau)\,d\tau \right)^{\frac{1}{q+1}} \tag{5}$$

Similarly the memcapacitor can be derived from the relation

$$\begin{aligned} & q_c(t) = c_m\,(x, v, t)\,\omega(t) \\ & \dot{x} = f\,(x, v, t) \end{aligned} \tag{6}$$

where $q_c(t)$ is quantity of charge at time $t$, $x$ is the correspondence internal state variable and $c_m$ is memcapacitor. The voltage across memcapacitor [36] is given by the relation

$$v(t) = c_m^{-1}\,(x, q_c, t)\,q_c(t) \tag{7}$$

$c_m^{-1}$ is inverse memcapacitance. Equations (6) and (7) can be simplified to a generalized forms as,

$$q(t) = c_m D^q[v\,(\tau)\,d\tau]v(t)\,,\ q < 0 \tag{8}$$

$$v(t) = c_m^{-1} D^q[q\,(\tau)\,d\tau]q(t)\,,\ q < 0 \tag{9}$$

Equation (8) is the voltage controlled memcapacitance and Equation (9) is the charge controlled memcapacitance. Using Riemann - Lioville Theorem the fractional order model of (8) and (9) can be derived as

$$v(t) = \frac{c_m^{-1}}{\Gamma(q)} \left[ \int_0^t (t-\tau)^{q-1} q(\tau)\, d\tau \right] q(t) \tag{10}$$

$$q(t) = \frac{c_m}{\Gamma(q)} \left[ \int_0^t (t-\tau)^{q-1} v(\tau)\, d\tau \right] v(t) \tag{11}$$

Equation (10) shows the fractional order charge controller memcapacitor and (11) shows the fractional order voltage controlled memcapacitor.

There have been discussions on using memristor, memcapacitor, cubic nonlinear resistors, piecewise linear functions being used as the nonlinear element to generate chaos and hyperchaos [28, 29, 30, 31, 32, 33, 34]. Some special phenomena such as hidden attractors as well as coexisting attractors are also discussed in the literatures [32, 33, 34]. Fractional order memristor based nonlinear oscillators have also been proposed and investigated [25, 35, 36] where the authors derived fractional order from the integer order model. In our earlier discussion about using the fractional order memristor element directly [27] rather than deriving the fractional order model from integer order we used all the states of the oscillator as commensurate fractional order. In this paper we are focusing our discussions towards using the fractional order model to only the memcapacitor charge equation and the other states are kept as integer order. For this analysis we used one of our memcapacitor oscillator from our earlier work [36] and replaced the memcapacitor with fractional order memcapacitor as shown in Fig.1.

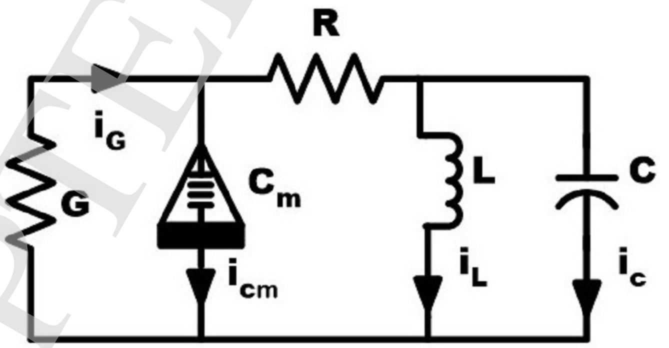


Figure 1: Fractional order memcapacitor based chaotic oscillator

$R$ is the resistance, $L$ is the inductances, $G$ is the conductance and $C$ is the capacitance. $C_m$ is the memcapacitor as discussed in [36]. The current flowing through the circuit are $i_G, i_R, i_{C_m}, i_L$. Applying Kirchhoffs law to the circuit shown in figure 1,

$$\begin{aligned} R\frac{d^q q_{C_m}}{dt} &= V_C + \left(G - \frac{1}{R}\right)(\alpha - \beta\sigma)\, q_{C_m} \\ C\frac{dv_c}{dt} &= \frac{1}{R}\left((\alpha - \beta\sigma)\, q_{C_m} - V_C\right) - i_L \\ L\frac{di_L}{dt} &= V_c \end{aligned} \tag{12}$$

where $q_{C_m}$ is the charge of the memcapacitor and $\sigma = D^q q_{C_m}$, $q < 0$. Integrating equation (12) with respect to time and using the flux relation $\phi(t) = \alpha\sigma + \frac{1}{2}\beta\sigma^2$

$$\begin{aligned} R\frac{d^q\sigma}{dt} &= \phi_c + \left(G - \frac{1}{R}\right)\left(\alpha\sigma - \frac{1}{2}\beta\sigma^2\right) \\ C\frac{d\phi_c}{dt} &= \frac{1}{R}\left(\alpha\sigma - \frac{1}{2}\beta\sigma^2 - \phi_c\right) - q_L \\ L\frac{dq_L}{dt} &= \phi_c \end{aligned} \tag{13}$$

where $q_L = \int i_L(t)dt$, $\phi_c = \int V_c(t)dt$ and $\sigma = D^q q_{C_m}$, $q < 0$. Let us define the dimensionless states of the system as $x = \sigma_m, y = \phi_c, z = -Rq_L$ and the parameters are defined as $a = C\alpha(RG - 1)$, $b = \frac{C\beta}{2}(RG - 1)$, $c = C$, $d = \alpha$, $e = \frac{\beta}{2}$, $f = -\frac{R^2C}{L}$ and for the values of $L = 0.13H$, $C = 3.57F, G = 2.1, R = 211\Omega, \alpha = 0.7F^{-1}$ and $\beta = 0.8F^{-1}c^{-1}s^{-1}$ the FMC system shows chaotic oscillations and the corresponding parameter values are derived as $a$ = - 1.638, $b$ = - 0.936, $c$ = 4.5, $d$ = 0.7, $e$ = 0.4 and $f$ = - 1.75. The dimensionless model of the fractional memcapacitor chaotic (FMC) system is given in (14) The Initial conditions are chosen as [0.1,0.1,0.1]. Figure 2 shows the 2D phase portraits of the system (14).

$$\begin{aligned} D^q x &= ax + bx^2 + cy \\ \dot{y} &= dx + ex^2 - y + z \\ \dot{z} &= fy \end{aligned} \tag{14}$$

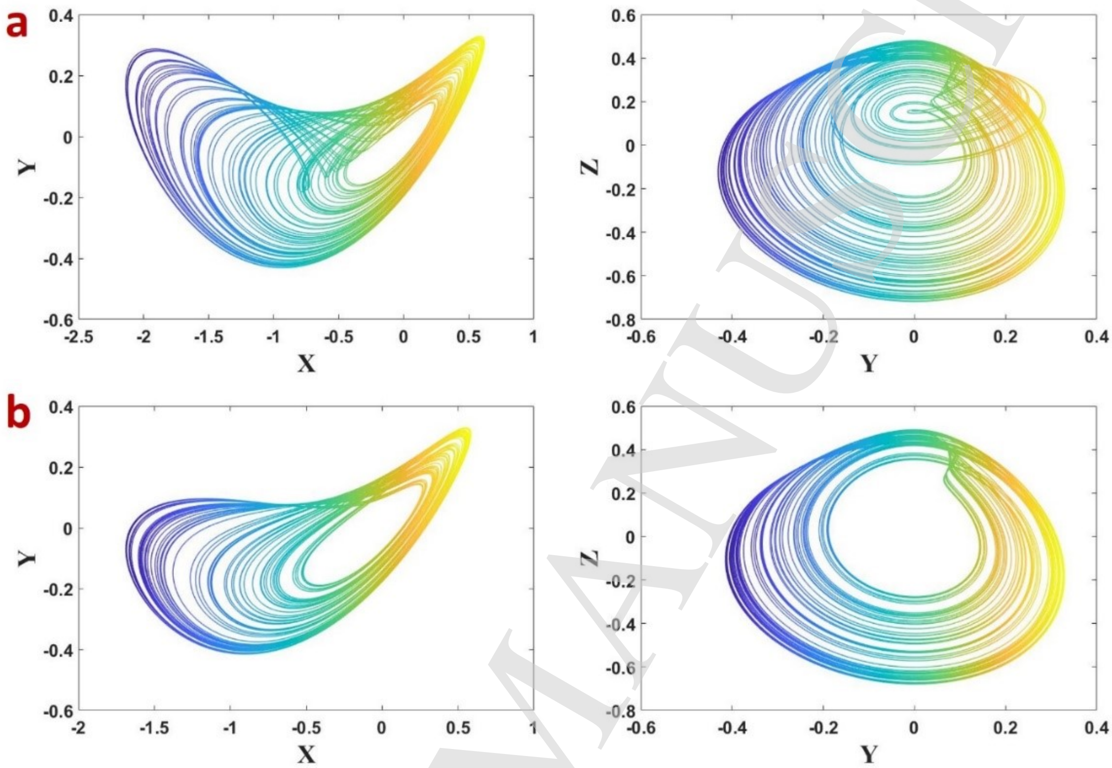


Figure 2: 2D phase portraits of the FMC system for a) q=0.99 ;b) q=0.9 for initial conditions [0.1, 0.1, 0.1]

## 3. Dynamic analysis of FMC system

In this section we derive the various dynamical properties of the FMC system like the equilibrium points, Eigen values, stability of equilibrium, Lyapunov exponents (LEs) and bifurcation plots.

### 3.1. Equilibrium Points

By equating $\dot{X} = 0$, the FMC system (14) shows two equilibrium points $E_1 = [0, 0, 0]$ and $E_2 = [-\frac{a}{b}, 0, \frac{adb-a^2e}{b^2}]$. The Jacobian matrix of the FMC system (14) is

$$J(X) = \begin{vmatrix} a+2bx & c & 0 \\ d+2ex & -1 & 1 \\ 0 & f & 0 \end{vmatrix} \tag{15}$$

The characteristic equation of the system are given in Table 1,

Table 1: The characteristic equation of the system

| Equilibrium points | Characteristic polynomial |
|---|---|
| $E_1 = [0,0,0]$ | $\lambda^{29} + \lambda^{21} + (2-a)\lambda^{20} + \lambda^{19} + 2\lambda^{12} + (2-2a)\lambda^{11} + (1-cd-a)\lambda^{10}$ $-f\lambda^9 + \lambda^3 + (1-a)\lambda^2 + (-a-f-cd)\lambda + af$ |
| $E_2 = [-\frac{a}{b}, 0, \frac{adb-a^2e}{b^2}]$ | $\lambda^{29} + \lambda^{21} + \left(\frac{2b+ab}{b}\right)\lambda^{20} + \lambda^{19} + 2\lambda^{12} + \left(\frac{2b+2ab}{b}\right)\lambda^{11}$ $+ \left(\frac{b+ab+2ace-bcd}{b}\right)\lambda^{10} - f\lambda^9 + \lambda^3 + \left(\frac{b+ab}{b}\right)\lambda^2$ $+ \left(\frac{ab-bf+2ace-bcd}{b}\right)\lambda - af$ |

The characteristic polynomial at the equilibrium $E_1$ and for the parameter values $a$ = - 1.638, $b$ = - 0.936, $c$ = 4.5 $d$ = 0.7,$e$ = 0.4 and $f$ = - 1.75. are $\lambda^{29} + \lambda^{21} + 3.638\lambda^{20} + \lambda^{19} + 2\lambda^{12} + 5.276\lambda^{11} - 0.512\lambda^{10} + 1.75\lambda^9 + \lambda^3 + 2.638\lambda^2 + 0.238\lambda + 2.8665$ and for the equilibrium point $E_2$, the characteristic polynomial is $\lambda^{29} + \lambda^{21} + 0.362\lambda^{20} + \lambda^{19} + 2\lambda^{12} - 1.276\lambda^{11} + 2.512\lambda^{10} + 1.75\lambda^9 + \lambda^3 - 0.638\lambda^2 + 3.262\lambda - 2.8665$ . The necessary condition for the FMC system to exhibit chaotic oscillations is $\frac{\pi}{2M} - \min_i(|\arg(\lambda_i)|) > 0$, where $M$ the LCM of the fractional orders. For equilibrium $E_1$ the FMC has a real root for the characteristic equation (-0.9858) whose argument is '0' and hence $\frac{\pi}{2M} - 0 = 0.157 > 0$ and similarly for equilibrium $E_2$ the FMC has a real root 0.7582, whose argument is '0' with $\frac{\pi}{2M} - 0 = 0.157 > 0$ which confirms that the FMC systems shows chaotic oscillations [39, 43, 47] as its integer order model [36].

### *3.2. Lyapunov Exponents and Kaplan-Yorke Dimension*

The Lyapunov exponents (LEs) of the FMC system are derived using the Wolfs algorithm [48] by using the fractional order predictor-corrector [49, 50] solver fde12 [51] in place of the ode solvers [52]. The fde12 solver is modified accordingly so that we could use the fractional orders for one state variable whereas the other two states remains in the integer order. The Lyapunov exponents of the FMC system are numerically found as $L_1 = 0.126$, $L_2 = 0$, $L_3 = -2.231$ .Since there is a positive Lyapunov exponent it is clear that the FMC system (14) is a chaotic attractor. The Kaplan-Yorke dimension of the FMC system is calculated as $D_{KY} = 2.06$.

*3.3. Bifurcation*

The dynamic behavior of the FMC system for change in parameter is investigated using the bifurcation plots. The parameter $b$ is chosen as the control parameter while the other parameters are taken as $a$ = - 1.638, $c$ = 4.5, $d$ = 0.7,$e$ = 0.4 and $f$ = - 1.75. The initial condition for the first iteration is $[0.1, 0.1, 0.1]$ which is changed to the end values of states $x, y, z$ in every iteration and local maxima of the state variable is plotted. Fig.3a shows the bifurcation of the FMC system with parameter $b$ for the fractional order $q = 0.99$.

The FMC system multiple chaotic regions for $0.948 \leq b \leq -0.9$, $-0.897 \leq b \leq -0.865$, $-0.832 \leq b \leq -0.788$, $-0.785 \leq b \leq -0.753$, $-0.751 \leq b \leq -0.745$ and $-0.743 \leq b \leq -0.737$. The FMC system take a period halving exit from chaos. These chaotic regions are confirmed from the positive Lyapunov exponents (LEs) shown in Fig.3b.

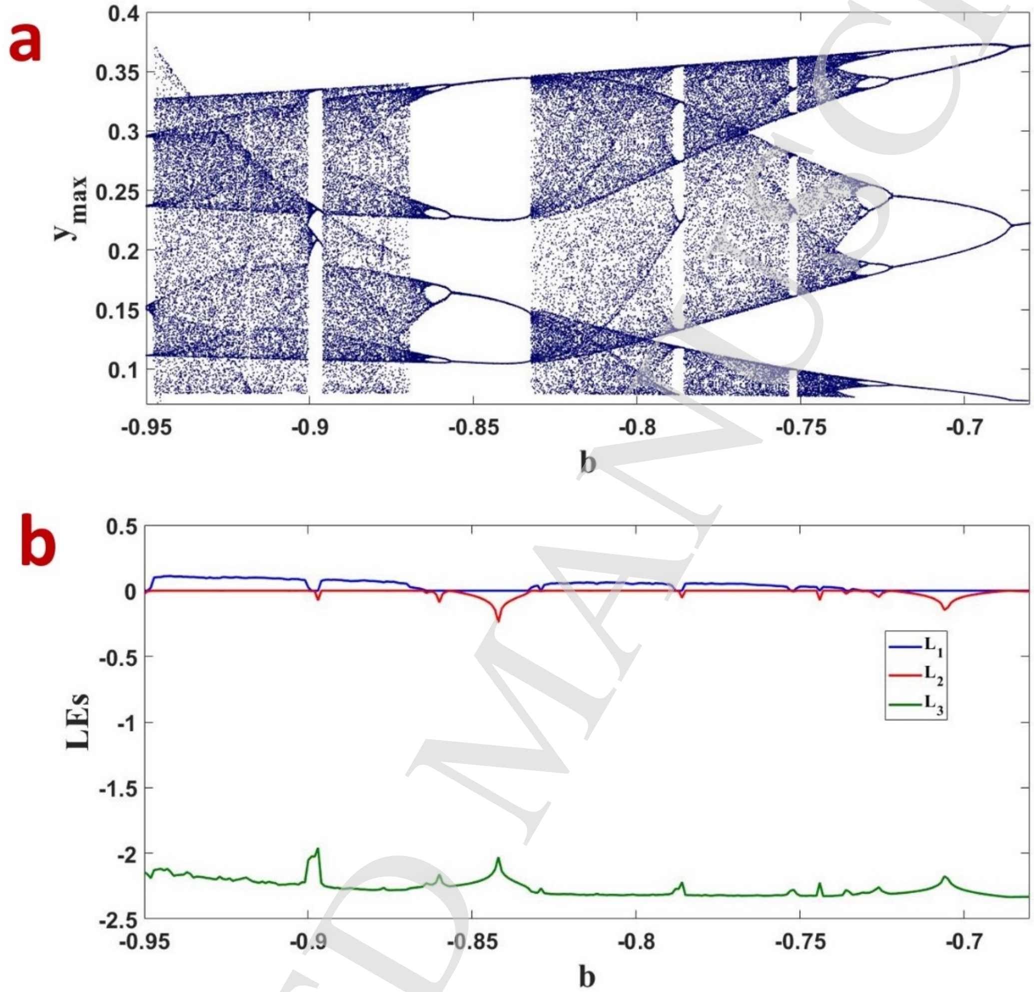


Figure 3: a) Bifurcation of the FMC system with parameter using forward continuation (parameter increased with reinitializing of initial conditions to end values of state variables) with initial conditions for the first iteration taken as $[0.1, 0.1, 0.1]$ , parameters $a$ = - 1.638, $b$ = - 0.936, $c$ = 4.5, $d$ = 0.7,$e$ = 0.4 $f$ = - 1.75. and fractional order $q = 0.99$; b) The corresponding LEs.

Similarly bifurcation of the FMC system with fractional order is shown in Fig.4. The system parameters are $a$ = - 1.638, $b$ = - 0.936, $c$ = 4.5, $d$ = 0.7,$e$ = 0.4 and the initial condition are taken as $[0.1, 0.1, 0.1]$ . The FMC system shows multiple chaotic regions for $0.8623 \leq q \leq 0.9173$, $0.9206 \leq q \leq 0.9321$ and $0.9478 \leq q \leq 1$

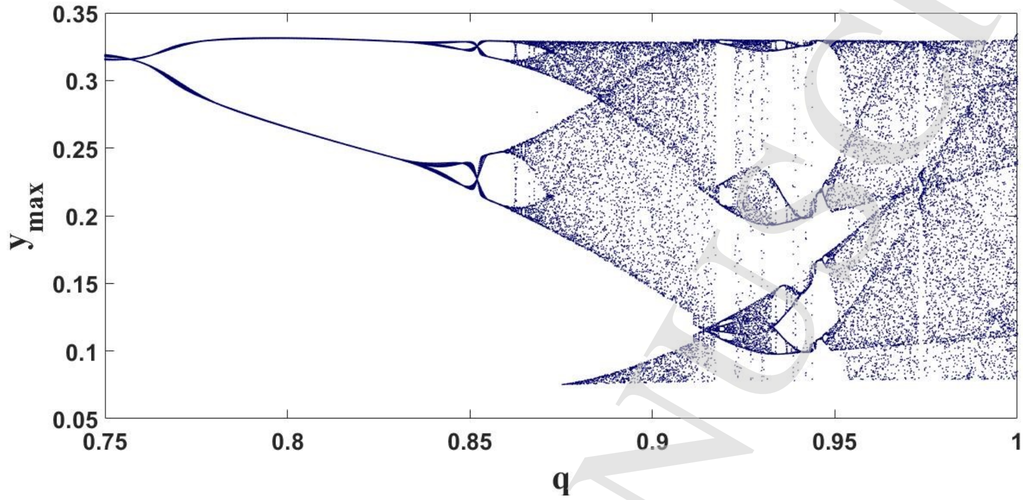


Figure 4: a) Bifurcation of the FMC system with fractional order $q$ using forward continuation with initial conditions for the first iteration taken as$[0.1, 0.1, 0.1]$ and parameters $a$ = - 1.638, $b$ = - 0.936, $c$ = 4.5, $d$ = 0.7,$e$ = 0.4, $f$ = - 1.75.

The basin of attraction of the FMC system is given in Fig.5 for two different initial condition of $y$ with Fig.5a showing the cross section of $x - z$ plane at $y = 0$ and Fig.5b showing the cross section of $x - z$ plane at $y = -0.1$.

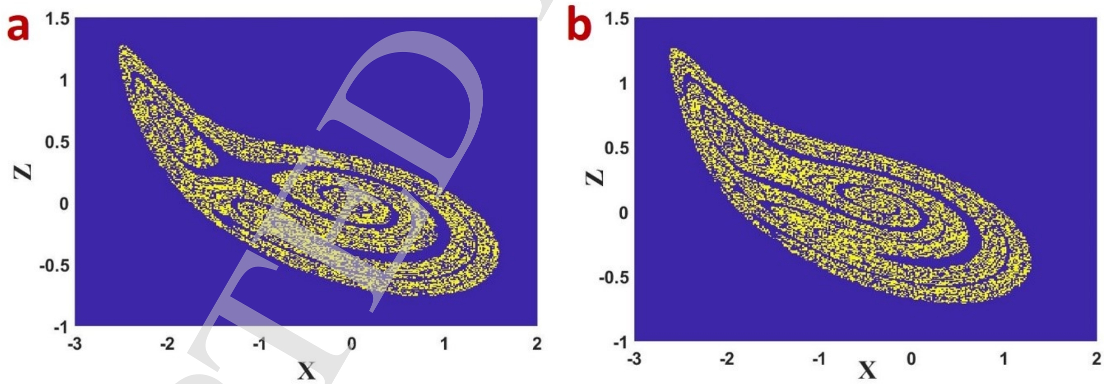


Figure 5: Cross section of the basin of attraction in the $x - z$ plane for a) $y = 0$ ; b) $y = -0.1$

## 4. Circuit Implementation of FMC system

The RiemannLiouville definition of fractional order integral is

$$\mathcal{I}^q f(t) \triangleq \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau)\, d\tau, \quad t>0,\ q \in R^+ \tag{16}$$

where $q$ is the fractional order, $\mathcal{I}^q$ is the $q^{th}$ fractional order Riemann-Liouville integral and $\Gamma(q)$ is the gamma function. If the initial values of $f(t)$ are equal to zero, then the fractional order integral given Eq.16 can be represented in frequency domain as:

$$\mathcal{L}\{\mathcal{I}^q f(t)\} = \frac{1}{s^q} F(s) \tag{17}$$

where $F(s)$ is the Laplace transform of function $f(t)$. Hence it can be said that the transfer function of fractional order integrator is

$$H(s) = \frac{1}{s^q} \tag{18}$$

However, the transfer function in Eq.18 cannot be realized directly with circuit elements. To overcome this setback, Charef et.al [46] proposed a method approximate the transfer function in Eq. 18 that the approximated transfer function can be realized with RC ladder network.
The transfer function in Eq.18 has a slope of $-20q$ dB/decade and can be approximated with zig-zag lines that have slopes of 0 dB/decade and 20 dB/decade. According to [46] the transfer function of the fractional order integrator can be approximated as:

$$H(s) = \frac{1}{s^q} \approx \frac{1}{(1+\frac{s}{p_t})^q} \approx \frac{\prod_{i=0}^{N-1}(1+\frac{s}{z_i})}{\prod_{i=0}^{N}(1+\frac{s}{p_i})} \tag{19}$$

Where $p_t$ is the corner frequency (or $1/p_t$ is the relaxation time), $z_i$ and $p_i$ are the zeros and poles of the approximated transfer function respectively and calculated as:

$$\begin{aligned} p_0 &= p_t 10^{y/20q} \\ z_i &= (ab)^i a p_0 \\ p_i &= (ab)^i p_0 \end{aligned} \tag{20}$$

a and b in Eq.20 are calculated as:

$$\begin{aligned} a &= \tfrac{z_{n-1}}{p_{n-1}} = 10^{y/10(1-q)} \\ b &= \tfrac{p_n}{z_{n-1}} = 10^{y/10q} \end{aligned} \tag{21}$$

y in Eq.20, 21 represents the maximum error in dB between the actual line and the approximated zig-zag line. N in Eq.19 can be calculated as:

$$N = Integer\left(\frac{\log\left(\frac{w_{max}}{p_0}\right)}{\log\left(ab\right)}\right) + 1 \tag{22}$$

It can be seen from Eq.22 that the value of the N is dependent on the frequency band, the maximum error between the actual line and the approximated line and the fractional order. All these steps given in Eq.19-22 are described in [46].
Ahmad and Sprott in their study [40], calculated approximated transfer function of fractional orders between the interval [0.1 0.9] with 0.1 step for 2 dB and 3 dB maximum errors using the algorithm given in [46].
In this study, the fractional order memcapacitor based chaotic system has been investigated for two different fractional order: $q$=0.9 and $q$=0.99. For the case when the fractional order $q$=0.9, the approximated transfer function with maximum error $y$=2 dB given in [40] is used. The approximated transfer function of $1/s^{0.9}$ is given in Eq.23.

$$\frac{1}{s^{0.9}} \approx \frac{2.2675\left(s+1.2922\right)\left(s+215.4\right)}{\left(s+0.01292\right)\left(s+2.154\right)\left(s+359.4\right)} \tag{23}$$

For the case when the fractional order $q$=0.99, the approximated transfer function is calculated by using described steps given in [46]. In the calculation, the corner frequency $p_t$ is chosen as 0.01 rad/sec and the maximum frequency $w_{max}$ is chosen as 100 rad/sec as in study [40] and the maximum error y is chosen as 0.2 dB. The approximated transfer function of $1/s^{0.99}$ is given in Eq.24.

$$\frac{1}{s^{0.99}} \approx \frac{1.073\left(s+1.0235\right)\left(s+107.2\right)}{\left(s+0.0102\right)\left(s+1.072\right)\left(s+112.3\right)} \tag{24}$$

For fractional order integrator the circuit shown in Fig.6 is used.

The transfer function of the fractional integrator given in Fig.6 is

$$\frac{V_o}{V_i} = -\frac{Z\left(s\right)}{R} \tag{25}$$

If the minus sign is dropped in Eq.25 and the value of R is selected as 1, then the impedance $Z(s)$ is equal to the approximated transfer function.

$$Z\left(s\right) \approx \frac{1}{s^q} \tag{26}$$

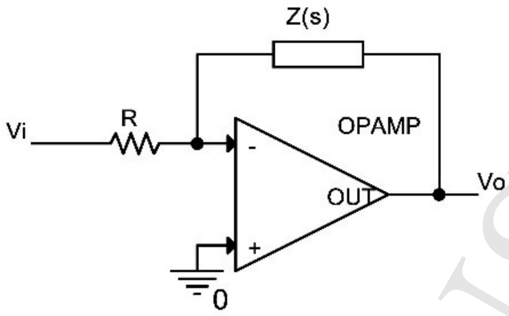


Figure 6: The fractional order integrator.

If the both nominator and denominator of the approximated transfer functions given in Eq.23 and 24 are examined, it can be said that both nominators and denominators are a positive real function and difference between degrees of the nominator and denominator polynomials in the both transfer functions is not greater than one, then the both approximated transfer functions given in Eq.23 and 24 are a real positive function. Hence, the impedance $Z(s)$ for the both transfer function can be realized any type of passive network (RC, RL).

In this study, we used Foster I method to synthesize the impedance $Z(s)$ with RC ladder network. In this method, the impedance is synthesized by partial fraction expansion of the impedance function. Since the degrees of the both denominator polynomials are 3 and the degrees of the both nominator polynomials are 2, the partial fraction expansion form of the both impedance functions $Z(s)$ will have the following form:

$$Z\left(s\right) = \frac{k_1}{s+\sigma_1} + \frac{k_2}{s+\sigma_2} + \frac{k_3}{s+\sigma_3} \tag{27}$$

The implementation of $Z(s)$ given in Eq.27 can be seen in Fig. 7. In the figure the value of capacitors is calculated as $C_i = \frac{1}{k_i}$ and the value of resistors is calculated as $R_i = \frac{k_i}{\sigma_i}$ .

The calculated resistors and capacitors values are given in Table 2 for both the fractional order. The value a in Table 2 is the integration coefficient.
Table 2: The resistors and capacitors values of the fractional order integrators for the fractional orders $q = 0.9$ and $q = 0.99$.

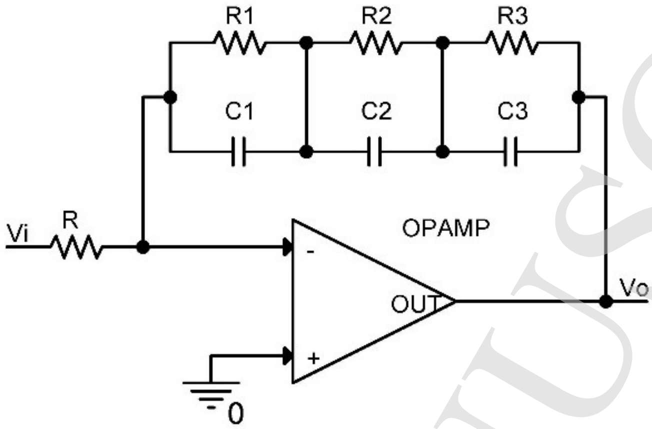


Figure 7: The fractional order integrator circuit synthesize from the transfer functions given in Eq.23 and 24.

Table 2: The resistors and capacitors values of the fractional order integrators for the fractional orders q=0.9 and q=0.99.

| | $q = 0.9$ | $q = 0.99$ |
|---|---|---|
| $R\,(\Omega)$ | 1/a | 1/a |
| $R_1\,(\Omega)$ | 0.00253 | $4.37e^{-4}$ |
| $C_1\,(F)$ | 1.098 | 20.367 |
| $R_2\,(\Omega)$ | 0.253 | 0.0439 |
| $C_2\,(F)$ | 1.833 | 21.23 |
| $R_3\,(\Omega)$ | 62.922 | 95.76 |
| $C_3\,(F)$ | 1.232 | 1.0238 |

If Table 2 is examined, the values of both resistors and capacitors are not much suitable for practical applications. However, these values can be shifted to a more reasonable range with magnitude scaling. Magnitude scaling is process of scaling all the impedances in a network with the same factor that frequency response of the network remains unchanged. In this study, the magnitude scaling factor is chosen as $k_m = 4.10^5$.

After magnitude scaling process, it is needed frequency scaling in order to obtain phase portraits of the fractional order chaotic system with an analog oscilloscope. The frequency scaling factor chosen as $k_f = 2500$. In the frequency scaling process of an RC network, the value of capacitors decreases with the scaling factor while that of resistors remain unchanged.

The values of the resistors and capacitors are given in Table 3 after the both magnitude and frequency scaling processes. The value a in Table 3 is the integration coefficient. Table 3: The resistors and capacitors values of the fractional order integrators for the fractional orders $q = 0.9$

and $q = 0.99$ after the scaling processes.

Table 3: The resistors and capacitors values of the fractional order integrators for the fractional orders q=0.9 and q=0.99.

| | $q = 0.9$ | $q = 0.99$ |
|---|---|---|
| $R\,(k\Omega)$ | 400/a | 400/a |
| $R_1\,(k\Omega)$ | 1.012 | 0.1748 |
| $C_1\,(nF)$ | 1.098 | 20.367 |
| $R_2\,(k\Omega)$ | 101.2 | 17.56 |
| $C_2\,(nF)$ | 1.833 | 21.23 |
| $R_3\,(k\Omega)$ | 25169 | 38304 |
| $C_3\,(nF)$ | 1.232 | 1.0238 |

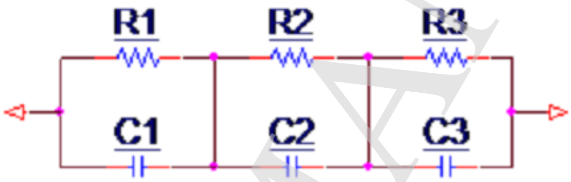


Figure 8: Fractional order impedance for transfer functions given in Eq. 23 and 24. (Diyi-Chen Model [53])

The circuit model of the fractional order memcapacitor based chaotic system is given in Fig.9. The fractional order impedance shown in Fig.8 is consists of RC ladder network shown as in Fig. 8. The value of resistors and capacitors in Fig.8 is given in Table 2 for the fractional orders $q = 0.9$ and $q = 0.99$.

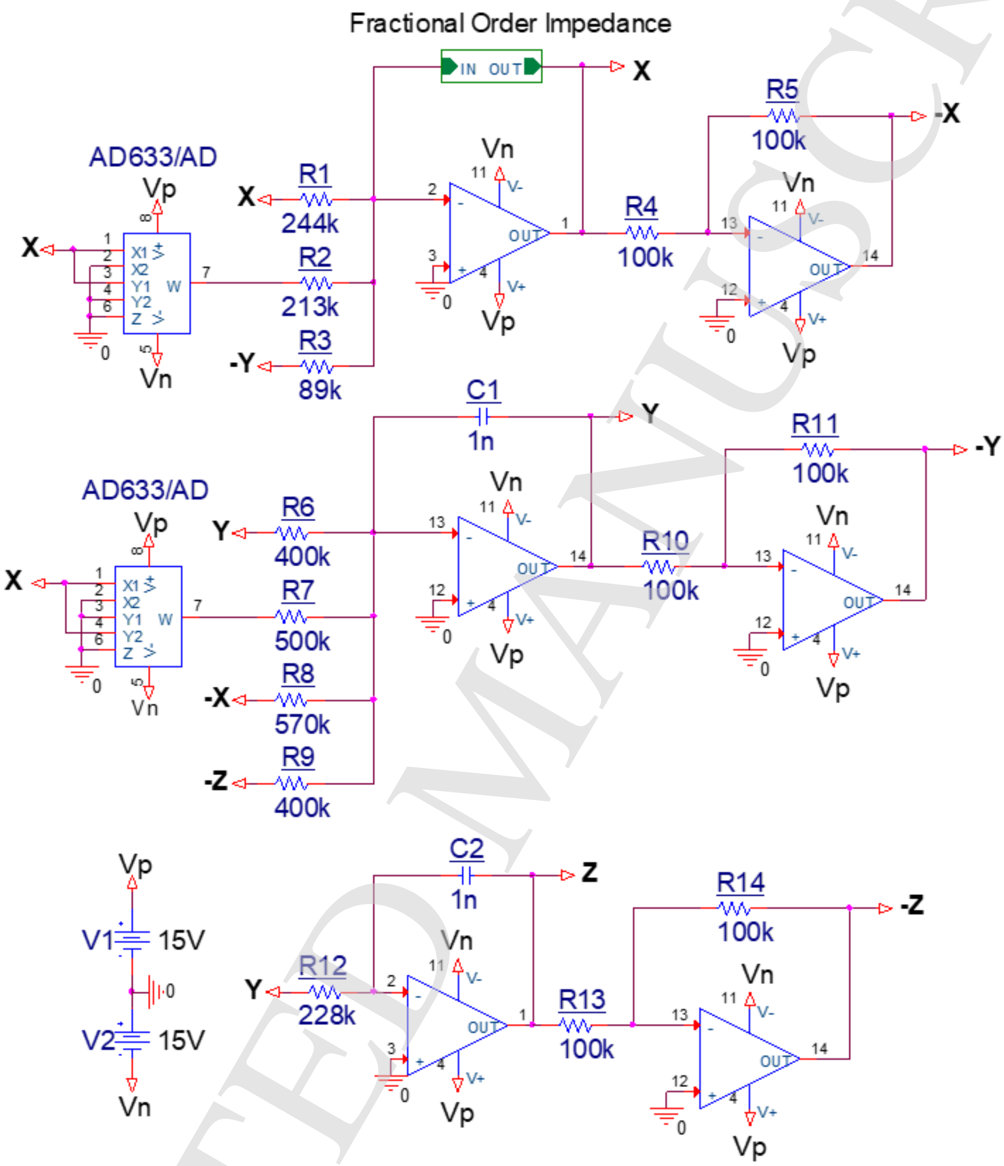


Figure 9: The circuit implementation of the fractional order memcapacitor based chaotic system.

The phase portraits of FMC system for the both fractional order $q = 0.9$ and $q = 0.99$ obtained via an oscilloscope are given in Fig.10.

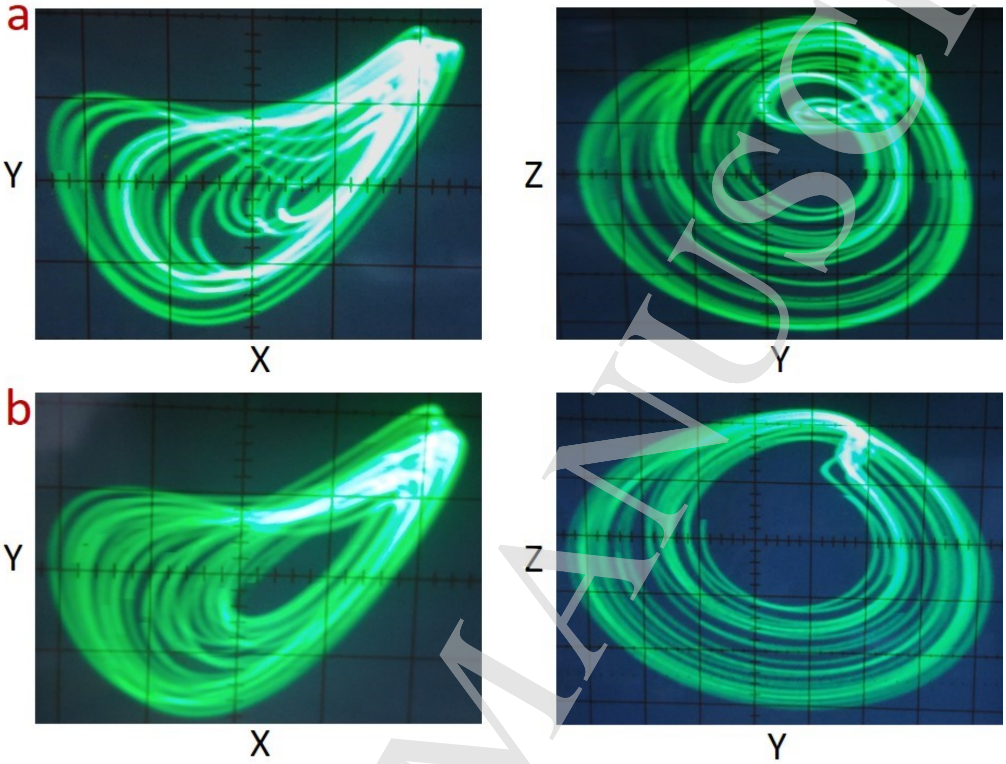


Figure 10: 2D phase portraits of the FMC system obtained from oscilloscope for a)$q = 0.99$;b)$q = 0.99$ for initial conditions $[0.1, 0.1, 0.1]$, (Volt/Div=0.5V).

The electronic circuit model of FMC system is realized using the electronic card in [54]. The picture of the circuit realization of FMC system is given in Fig.11.

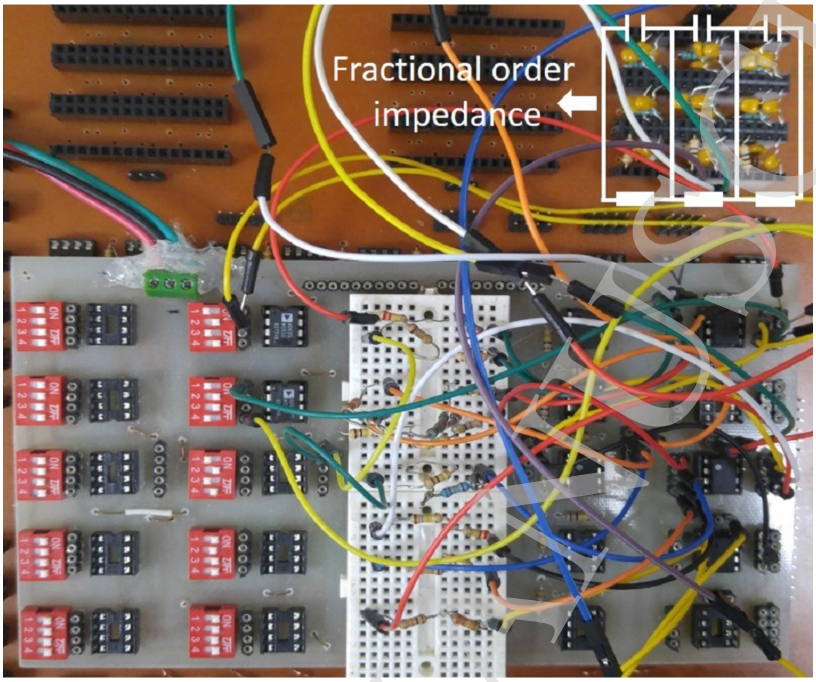


Figure 11: The circuit realization of FMC system.

## 5. Conclusion

A new chaotic oscillator with a fractional order memcapacitor is proposed and analysed. The dynamical properties of the chaotic oscillator are investigated with equilibrium points, Eigen values, Lyapunov exponents and bifurcation plots. The proposed FMC system is then realised in hardware using off the shelf components. The experimental results confirm that the proposed oscillator is hardware realisable. For the future research on similar areas, various memristor models such as discontinuous memristor, exponential flux controlled memristor, second order memristor, etc, can be considered. Also, the proposed novel FMC system can be in used many science and engineering fields.

**Acknowledgment**

This work has been supported by Sakarya University Scientific Research Projects Unit (Project Number: 2017-09-00-010).

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