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Comparison of metaheuristic optimization algorithms with a new modified deb feasibility constraint handling technique

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Abstract: In this study, the modification of the Deb feasibility method is considered to solve the constrained optimization problems. In the developed modified Deb feasibility constraint method, the third rule in its procedure was revised in order to increase the performance of the Deb feasibility constraint handling method. The innovation in the method is based on generating a new individual by using both possible solutions that violate the constraints in the method used for solving the problem. In detail, discussions were given about the application and usefulness of six constrained handling techniques. Furthermore, genetic algorithm, particle swarm optimization, Harris hawks optimization, whale optimization algorithm, grey wolf optimization and sine cosine algorithms were applied to both various benchmark functions and also different engineering application problems such as pressure vessel design, welded beam design, speed reducer design and active filter design. Overall the experimental results show that modified Deb feasibility constraint handling technique is more robust and efficient than Deb feasibility technique and most of the other constraint handling techniques.

Key words: Constraint handling methods, metaheuristic algorithms, Deb feasibility

1. Introduction

Metaheuristic algorithms are used in solving optimization problems in many fields including engineering, business, and science [1]. The main purpose of the solution process in optimization problems is to minimize or maximize the performance, duration, efficiency, and productivity parameters. Most of the optimization problems in real-world contain some constraints defined on decision variables [2–4].

Traditional optimization methods might not successfully address optimization problems due to discontinuities and nonlinearities in design spaces. Metaheuristic algorithms, on the other hand, are effectively applied in cases where traditional algorithms are inadequate to solve a wide variety of problems and applications. Most of the metaheuristic optimization algorithms work by mimicking biological processes, in particular, the evolution of species or swarm behavior [5, 6]. Some of the most recent multiobjective algorithms are ant colony algorithm (ACO) [7], artificial bee colony (ABC) [7], grey wolf optimizer (GWO) [8, 9], firefly algorithm (FA) [7], cuckoo search optimization (CS) [7, 10], bat algorithm (BA) [7], salp swarm algorithm (SSA) [11], whale optimization algorithm (WOA) [12] and Harris hawks optimization (HHO) [13].

Metaheuristic algorithms are global search techniques that are less likely to get stuck (to be caught) at optimization local optima. Moreover, these algorithms can adapt to different types of problems such as

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complex, nonlinear, and discontinued [7, 14]. Metaheuristic algorithms have been successfully complemented by constraint handling techniques to solve constrained problems, guiding the search process to feasible regions and ideally providing solutions that do not violate any constraint [15]. Besides, metaheuristic algorithms are efficient and can be easily adapted, if the constraints are properly identified in solving optimization problems. There are several ways of managing the problem constraints in a metaheuristic algorithm: penalty terms in the objective function of the problem, special representations and operators, separation of objectives and constraints and repair metaheuristic [16]. A proper constraint handling method when used in conjunction with a compatible metaheuristic algorithm can drive the search process towards global feasible optima by making use of the information present in the infeasible solutions [17]. Furthermore, in the literature, there are various constraint handling techniques such as death penalty, static penalty, dynamic penalty, barrier function, epsilon constraint handling, stochastic ranking, and Deb feasibility [18].

Most of the research conducted on constraint handling methods is focused on improving the performance and effectiveness of metaheuristic algorithms. Gandomi et al. increased the evaluation of viable solutions by indirectly addressing constraints and thus created a method of updating boundaries in the solution [14]. Babalik et al. modified the tree-seed algorithm using Deb rules to solve constrained optimization problems. In the tree-seed algorithm, Deb's objective function and violation of constraint properties are used for selecting the better individuals. They compared the performance of the developed algorithm with the results of GA, PSO and ABC [2]. In the study conducted by Miranda-Varela et al., the surrogate model and differential evolution were combined to approximate the objective function value and violation of constraint values. In this approach model, they worked with different constraint techniques [19]. In the study by Samanipour et al., the efficiency of nondominated constrained methods based on genetic algorithms were improved. They made repairs on solutions that affected the current generation constraint in the genetic algorithm and performed it on the NSGA-II method [5]. In Long's study, they tried a new restriction method on the genetic algorithm. In order to provide flexible solutions for constrained multipurpose problems, they took advantage of the Pareto boundary proximity and solution flexibility of the genetic algorithm [20]. In their study, He et al. compared constraint methods such as penalty methods, barrier functions, ϵ-constrained method, feasibility criteria, and stochastic ranking to address optimization problems. In addition, the pressure vessel design problem was solved by the flower pollination algorithm [18].

In this study, the modified Deb feasibility method was obtained by developing the third rule property of the classic Deb feasibility constraint method used in optimization problems. In the developed method, both possible solutions that violate constraints were combined to create a new possible solution. With this modification, it was aimed to increase the performance and effectiveness of the constraint handling method improved by Deb. So, the performances of death penalty, static penalty, dynamic penalty, barrier function, Deb feasibility and modified Deb feasibility on metaheuristic algorithms were tested and discussed. Constraint handling methods were adapted to GA, PSO, HHO, WOA, GWO and SCA. Various constrained optimization problems in the mathematical and engineering studies such as pressure vessel design, welded beam design, speed reducer design and active filter design were implemented to assess the performance of the methods statistically. The results of engineering design optimization problems are compared with algorithms in the literature.

The outline of the paper is as follows: Section 1 presents some general information and summarizes the studies in the literature. After that, unconstrained, constrained problems, and constraint handling methods are described in Section 2. In Section 3, details of the developed constraint handling method are presented. In Section 4, results of numerical analysis of benchmark problems and engineering applications are presented.

Lastly, conclusion is given in Section 5.

2. Preliminaries

2.1. General optimization

Optimization problems can be in different forms in many engineering applications. The objective (fitness) function to be optimized can be single or multidimensional, as well as being linear and/or nonlinear. The structure of the multidimensional (D-dimensional) unconstrained optimization problem is given in Equation (1). There are D variables in this problem and it is desired to determine the minimum value of the function $f(X)$ for the solution. This function may have either a local optima then that is global optima or multiple local optima then one of them is global optima. So, functions with one global optima are called unimodal and functions with multiple local optima are called multimodal [6, 8, 12].

$$\min \quad f(X) \quad X = [x_1, x_2, .., x_D]^T \in \mathbb{R}^D \tag{1}$$

The problem becomes a constrained optimization problem when equality and/or inequality constraints need to be satisfied just like given as in Equation (2). While solving this problem, the solution must satisfies both $h(X)$ constraints, that is N equations, and $g(X)$ constraints, that is M inequalities.

$$\begin{array}{ccc} \min & f(X) & X = [x_1, x_2, .., x_D]^T \in \mathbb{R}^D \\ & h_i(X) = 0 & (i = 1, 2, ..., N) \\ & g_j(X) \leq 0 & (j = 1, 2, ..., M) \end{array} \tag{2}$$

In order to solve such problems, there are many different techniques that can work effectively. Unfortunately, although there exist various techniques, those techniques all could not guarantee to find global optimum among the local optimas at some difficult problems [7, 10, 21]. Therefore, scientists have been continuing to research to find out more effective algorithms and methods.

2.2. Constraint handling methods

2.2.1. Death penalty method (DP)

DP method is probably one of the easiest and most applicable constraint handling methods in the literature. Among the possible solutions, if all solutions remain within the feasible region, then function value is assigned. If a solution that does not satisfy at least one constraint, that is, the solution is in the infeasible region, then a very high value is assigned due to the constraint violation. Therefore, the objective function created can be expressed as in Equation (3).

$$\varphi(X) = \begin{cases} f(X) & \textbf{feasible} \\ K - \sum_{i=1}^{s} \frac{K}{M} & \textbf{infeasible} \end{cases} \tag{3}$$

In Equation (3), s represents the number of satisfied constraints, and K represents a very large constant that is assigned as 10^{10} so that the constraints remain in the feasible region. In this method, when the constraints are violated, the violations are evaluated with the same error, regardless of the value of the objective function. That is, when the constraints are violated; the magnitude of the violations and the value of the objective function are not important.

2.2.2. Static penalty method (SP)

In the SP constraint handling method, each violation value is punished by multiplying it with a fixed number. In this way, solutions are tried to be moved to the feasible region. To ensure this, the objective function is arranged as in Equation (4). In this equation, λ and μ parameters are fixed and can be chosen between $[1, \infty]$. However, in this study, values of λ and μ parameters are taken as 10^6.

$$\varphi(X) = f(X) + \lambda \sum_{i=1}^{N} |h_i(X)| + \mu \sum_{j=1}^{M} \mathbf{max}\{0, g_j(X)\} \tag{4}$$

2.2.3. Dynamic penalty method (DynP)

In the DynP, each constraint that violates the boundaries is punished again. During this process, the penalty coefficient $\lambda(t)$ is dynamically increased depending on the iteration number (t). The objective function for this process is arranged as in Equation (5).

$$\varphi(X) = f(X) + \lambda(t) \left(\sum_{i=1}^{N} (h_i(X))^2 + \sum_{j=1}^{M} (\mathbf{max}\{0, g_j(X)\})^2 \right) \tag{5}$$

Considering the studies in the literature, it is believed that changing the β variable depending on iterations may be an advantage [20, 21]. This penalty coefficient is also updated with $\lambda(t) = (\alpha t)^{\beta}$. The parameters of this process used in this study were taken as $\alpha = 0.5$ and $\beta = 1.2$. However, the very high objective function encountered in simulation studies showed that this error function was ineffective. Therefore, this structure was developed in this study and used in the form of $\lambda(t) = 10^3(1 + \alpha t)^{\beta}$ and the results were obtained in this way.

2.2.4. Barrier penalty method (BP)

Equality constraints can be provided by Lagrangian multipliers, while inequality constraints can be achieved by different methods. This is accomplished by adding functions that have a greater error or generate an infinite error as the solutions converge to the unfeasible region to the objective function. For this, either Equations (6) or (7) suggested in the literature can be used. When iteration increases during this process, parameter of μ converges to zero, that is, $t \to \infty$ and $\mu(t) \to 0$. Besides, the structure of $\mu(t)$ can be chosen in the form of $\mu(t) = 1/t$ or $\mu(t) = 1/\sqrt{t}$.

$$\varphi(X) = f(X) + \mu(t) \sum_{j=1}^{M} -\mathbf{log}(g_j(X)) \tag{6}$$

$$\varphi(X) = f(X) + \mu(t) \sum_{j=1}^{M} \frac{1}{g_j(X)} \tag{7}$$

2.2.5. Deb feasibility method (DF)

Deb suggested a set of rules for feasible and infeasible regions. In this method, firstly, the degree of constraint violation is calculated by using Equation (8) for each individual. If $\varphi(X)$ is greater than zero or a small value such as ϵ, then that causes constraint violations and the solution is in the infeasible region [22].

$$\varphi(X) = \sum_{i=1}^{N}(h_i(X))^2 + \sum_{j=1}^{M}(\mathbf{max}\{0, g_j(X)\})^2 \tag{8}$$

According to the result of the function calculated according to Equation (8), a selection is made from possible solutions. These rules improved by Deb are given below [22]:

- If two solutions are in the feasible region, then the solution with a smaller objective function value is preferred.
- If one solution is feasible and the other is not feasible, then the feasible solution is preferred.
- If two solutions are not feasible, then the solution with less constraint violation is preferred.

Thanks to this structure improved by Deb, all solutions that are not in the feasible region are tried to be moved or directed to the feasible region. The pseudocode of the DF improved by Deb is depicted in Algorithm 1. The steps of the DF are explained in almost every line of pseudocode of the DF. Also, in pseudocode, ϵ value given has crucial importance for equality constraints. This value was determined as 10^{-3} in the study.

Algorithm 1: Pseudocode for Deb feasibility method.

```
if φ(x_i) < ϵ and φ(x_j) < ϵ then // DF 1st rule: if both solutions are feasible then compare objective function values
    if f(x_i) < f(x_j) then // Choose smaller objective function value
        output is x_i // If f(x_i) is less than f(x_j) then choose x_i
    else
        output is x_j // If f(x_j) is less than f(x_i) then choose x_j
else if φ(x_i) < ϵ and φ(x_j) ≥ ϵ then// DF 2nd rule: choose feasible solution
    output is x_i// if φ(x_i) is feasible and φ(x_j) is not feasible then choose x_i
else if φ(x_i) ≥ ϵ and φ(x_j) < ϵ then// DF 2nd rule: choose feasible solution
    output is x_j // if φ(x_i) is not feasible and φ(x_j) is feasible then choose x_j
else // DF 3rd rule: both are in infeasible region, then choose less constraint violation
    if φ(x_i) < φ(x_j) then // Choose less contraint violation
        output is x_i // if φ(x_i) is less than φ(x_j) then choose x_i
    else
        output is x_j // if φ(x_j) is less than φ(x_i) then choose x_j
```

3. Modified Deb feasibility method (MDF)

The improved MDF is based on generating a new individual. This process is inspired by the bisection method. The essence of this hypothesis is to generate a new likely individual that less violates the constraints especially these might be linear, nonlinear, convex, nonconvex, continuous, or discontinuous. Specifically, modification of conventional DP is taken place at the 3rd rule. At the 3rd rule, if both individuals are in the infeasible region, then a new individual is generated using Equation (9) that might promote the performance. So, this modification allows finding a new solution in the feasible region for the constrained optimization problem. In order to express all the processes, the pseudocode of the MDF is depicted in Algorithm 2. Furthermore, steps of the MDF method are explained in almost every line of pseudocode of the MDF in Algorithm 2. Also, the ϵ value given in this pseudocode has crucial importance for equality constraints. This value was determined as 10^{-3} in the study.

$$x_0 = \frac{1}{2}(x_i + x_j) \tag{9}$$

Algorithm 2: Pseudocode for modified Deb feasibility method.

```
if φ(x_i) < ϵ  and  φ(x_j) < ϵ then // MDF 1st rule: if both solutions are feasible then compare objective function values
    if f(x_i) < f(x_j) then // Choose smaller objective function value
        output is x_i // If f(x_i) is less than f(x_j) then choose x_i
    else
        output is x_j // If f(x_j) is less than f(x_i) then choose x_j
else if φ(x_i) < ϵ  and  φ(x_j) ≥ ϵ then // MDF 2nd rule: choose feasible solution
    output is x_i // if φ(x_i) is feasible and φ(x_j) is not feasible then choose x_i
else if φ(x_i) ≥ ϵ  and  φ(x_j) < ϵ then // MDF 2nd rule: choose feasible solution
    output is x_j // if φ(x_i) is not feasible and φ(x_j) is feasible then choose x_j
else // MDF 3rd rule: both are in infeasible region, then generate a new individual and choose one that is the least constaint violation among (x_0, x_i, x_j)
    Generate a new x_0 // a new solution is generated by using Eq. 9
    if φ(x_i) < φ(x_j)  and  φ(x_i) < φ(x_0) then // Choose less contraint violation
        output is x_i // If φ(x_i) is less than φ(x_j) and φ(x_0) then choose x_i
    else if φ(x_j) < φ(x_i)  and  φ(x_j) < φ(x_0) then // Choose less contraint violation
        output is x_j // If φ(x_j) is less than φ(x_i) and φ(x_0) then choose x_j
    else
        output is x_0 // If φ(x_0) is less than φ(x_i) and φ(x_j) then choose x_0
```

4. Numerical experiments

Constraint handling methods in this study have been applied on GA, PSO, HHO, WHO, GWO and SCA metaheuristic algorithms proposed in recent years. Moreover, the algorithms were implemented under Matlab R2019a and SPSS Statistics 26 on a computer with a Windows 10 which is windows student version and has 8 GB RAM. The swarm sizes of each algorithm were determined as 50 and the maximum number of iteration as 1000. In addition, these algorithms and constraint handling methods were rerun 50 times to evaluate the statistical performance. Parameters of GA are determined such that simulated binary crossover (SBX) operator and parameter-based mutation operator (p_m) are adaptive [22]. R parameter is 10^{-3}. Subsequently, parameters of PSO are determined such that weight (w) is decreased linearly from 0.9 (w_{max}) to 0.4 (w_{min}) and correlation coefficients (c_1, c_2) are 2.0 [23]. For WOA, a parameter is chosen such that decreased from 2 to 0. r_1, r_2, p are random numbers in $[0,1]$. The others are the algorithm's inherent parameters determined with respect to [21]. For HHO, E_1 parameter is chosen such that decreased from 2 to 0. E_0 is a random number in $[-1,1]$. q and r are random numbers in $[0,1]$. The others are the algorithm's inherent parameters determined with respect to [13]. For GWO, a parameter is chosen such that decreased from 2 to 0. r_1, r_2, p are random numbers in $[0,1]$. The others are the algorithm's inherent parameters determined with respect to [9]. For SCA,

a parameter is chosen 2. r_1 parameter is linearly decreased from 2 to 0 as iteration number increases. r_2 is random number in $[0, 2\pi]$. r_3 is random number in $[0, 2]$. r_4 is random number in $[0, 1]$. The others are the algorithm's inherent parameters determined with respect to [24]. Furthermore, different benchmark problems and engineering application problems were used to evaluate the performance of constraint handling methods in their use with GA, PSO, WOA, GWO, HHO and SCA algorithms. In addition, the developed Deb feasibility algorithm improved as a state of the art has been compared with other methods.

4.1. Benchmark problems

There are difficult-to-solve problems that are very commonly used in evaluating and comparing swarm-based and evolutionary algorithms. These test functions, called benchmark problems, are used to evaluate the performance of algorithms. Many studies have been conducted in the literature to evaluate the metaheuristic algorithms used in this study on unconstrained optimization problems [8, 9, 12, 25]. Thus, as a state of art, solely constrained problems were given to evaluate constraint handling methods. Details of benchmark problems are given in Tables 1 and 2. Apart from problems in Table 2, the problems in Table 1 are given briefly and they exist in [22].

Table 1. Details of constraint benchmark problem between $F_1 - F_8$ [22].

	Number of equality constraints	Number of inequality constraints	Number of total constraints	Dimensions	F_{min}
F_1	0	2	2	2	13.59085
F_2	0	38	38	5	-1.9146
F_3	0	9	9	13	-15.00
F_4	0	6	6	8	7049.331
F_5	0	4	4	7	680.6306
F_6	0	6	6	5	-30665.7
F_7	3	0	0	3	0.05395
F_8	0	8	8	10	24.30621

Table 2. Details of constraint benchmark problem between $F_9 - F_{11}$.

Function equations	Dimension	F_{min}
$F_9(x) = (\sqrt{n})^n \prod_{i=1}^{n} x_i$ Subject to $h_1(x) = \sum_{i=1}^{n} x_i^2 - 1 = 0 \quad 0 \leq x_i \leq 1 \qquad i = 1, 2, ..., 10$	10	-1.0005
$F_{10} = (x_1 - 10)^3 + (x_2 - 20)^3$ Subject to $g_1(x) = (x_1 - 5)^2 + (x_2 - 5)^2 + 100 \leq 0$ $g_1(x) = -(x_1 - 6)^2 + (x_2 - 5)^2 + 82.81 \leq 0$ $13 \leq x_1 \leq 100 \qquad 0 \leq x_2 \leq 100$	2	-6961.813875
$F_{11} = x_1^2 + (x_2 - 1)^2$ Subject to $h_1(x) = x_2 - x_1^2 = 0 \quad -1 \leq x_1, x_2 \leq 1$	2	0.7499

Table 3. $F_1 - F_6$ Constrained benchmark problem optimizations min, median, max and violation/rerun results.

		Min	Median	Max	Violation /Rerun	Min	Median	Max	Violation /Rerun	Min	Median	Max	Violation /Rerun
F_1		GA				PSO				WOA			
	DP	1.3660E+01	6.4380E+01	2.6069E+02	0/50	**1.3584E+1**	**1.3584E+1**	**1.3584E+1**	0/50	**1.3584E+1**	1.3586E+01	1.3595E+01	0/50
	SP	1.3591E+01	1.3655E+01	1.5098E+01	0/50	1.3591E+01	1.3591E+01	1.3591E+01	0/50	1.3591E+01	1.3593E+01	1.3616E+01	0/50
	DynP	1.3591E+01	1.3605E+01	1.3776E+01	0/50	1.3591E+01	1.3591E+01	1.3591E+01	0/50	1.3591E+01	2.2156E+01	2.7048E+02	0/50
	BP	1.3591E+01	1.7722E+01	9.5231E+01	0/50	1.3591E+01	1.3591E+01	1.3591E+01	0/50	1.3591E+01	1.3593E+01	1.3609E+01	0/50
	DF	1.3584E+01	1.3723E+01	1.4425E+01	0/50	1.3591E+01	1.3591E+01	1.3591E+01	0/50	1.3591E+01	6.5161E+01	2.6915E+02	0/50
	MDF	1.3584E+01	1.3600E+01	2.0030E+01	0/50	1.3591E+01	1.3591E+01	1.3591E+01	0/50	1.3591E+01	1.8852E+01	1.3976E+02	0/50
		HHO				GWO				SCA			
	DP	**1.3584E+1**	1.3585E+01	1.3594E+01	0/50	1.3588E+01	1.3613E+01	1.3652E+01	0/50	**1.3584E+1**	1.3708E+01	1.3911E+01	0/50
	SP	1.3591E+01	1.3592E+01	1.3601E+01	0/50	1.3596E+01	1.3621E+01	1.3676E+01	0/50	1.3592E+01	1.3704E+01	1.3877E+01	0/50
	DynP	1.3591E+01	1.3592E+01	1.3598E+01	0/50	1.3592E+01	1.3612E+01	1.3682E+01	0/50	1.3595E+01	1.3727E+01	1.4037E+01	0/50
	BP	1.3591E+01	1.3592E+01	1.3612E+01	0/50	1.3594E+01	1.3616E+01	1.3655E+01	0/50	1.3603E+01	1.3743E+01	1.3996E+01	0/50
	DF	1.3591E+01	1.3596E+01	1.3649E+01	0/50	1.3592E+01	1.3617E+01	1.3644E+01	0/50	1.3591E+01	1.3594E+01	1.3602E+01	0/50
	MDF	1.3591E+01	1.3593E+01	1.3603E+01	0/50	1.3591E+01	1.3608E+01	1.3635E+01	0/50	1.3591E+01	1.3594E+01	1.3600E+01	0/50
F_2		GA				PSO				WOA			
	DP	-1.6528E+00	-1.3459E+00	-9.9803E-01	0/50	**-1.94054**	**-1.94044**	**-1.9018**	0/50	-1.6429E+00	-1.4068E+00	-1.1221E+00	0/50
	SP	-1.4317E+00	-1.1163E+00	-9.2426E-01	0/50	-1.9052E+00	-1.8851E+00	-1.4307E+00	0/50	-1.6477E+00	-1.3561E+00	-9.8903E-01	0/50
	DynP	-1.5734E+00	3.4200E+07	5.5700E+08	**8/50**	-1.9052E+00	-1.9039E+00	-1.9017E+00	0/50	-1.5826E+00	-1.3620E+00	-9.8728E-01	0/50
	BP	-1.5937E+00	-1.1155E+00	-9.2885E-01	0/50	-1.9052E+00	-1.8757E+00	-1.4307E+00	0/50	-1.8058E+00	-1.3440E+00	-9.9396E-01	0/50
	DF	-1.4978E+00	-1.1616E+00	-9.1794E-01	0/50	-1.9052E+00	-1.8938E+00	-1.4307E+00	0/50	-1.6098E+00	-1.2026E+00	-6.3553E-01	0/50
	MDF	-1.5764E+00	-1.1938E+00	-9.3111E-01	0/50	-1.9052E+00	-1.9041E+00	-1.9013E+00	0/50	-1.4507E+00	-1.1626E+00	-7.5630E-01	0/50
		HHO				GWO				SCA			
	DP	-1.7997E+00	4.1700E+96	2.0800E+98	**1/50**	-1.8861E+00	-1.6880E+00	-1.0405E+00	0/50	-1.6981E+00	-1.4114E+00	-1.1978E+00	0/50
	SP	-1.8223E+00	-1.6984E+00	-1.5685E+00	0/50	-1.7655E+00	-1.4827E+00	-1.0004E+00	0/50	-1.6736E+00	-1.4330E+00	-1.2294E+00	0/50
	DynP	-1.8547E+00	-1.7150E+00	-1.5721E+00	0/50	-1.8424E+00	-1.5889E+00	-1.0429E+00	0/50	-1.6877E+00	-1.4624E+00	-1.2353E+00	0/50
	BP	-1.8240E+00	-1.6964E+00	-1.5838E+00	0/50	-1.8104E+00	-1.4688E+00	-9.8751E-01	0/50	-1.7540E+00	-1.4506E+00	-1.1509E+00	0/50
	DF	-1.8494E+00	-1.7810E+00	-9.0340E-01	0/50	-1.7840E+00	-1.3561E+00	-8.4097E-01	**8/50**	-1.8131E+00	-1.4329E+00	-1.4001E+00	0/50
	MDF	-1.8578E+00	-1.7858E+00	-9.2676E-01	0/50	-1.7469E+00	-1.4116E+00	-8.5323E-01	0/50	-1.8514E+00	-1.7184E+00	-1.5004E+00	0/50
F_3		GA				PSO				WOA			
	DP	-1.4294E+01	-1.3050E+01	-6.5139E+00	0/50	-1.2002E+01	-7.7571E+00	-4.0000E+00	0/50	-1.2526E+01	-5.0005E+00	-2.9667E-03	0/50
	SP	1.3381E+01	1.3999E+01	1.4418E+01	0/50	-9.0000E+00	-6.3000E+00	-4.0000E+00	0/50	-1.2783E+01	-3.3173E+00	0.0000E+00	0/50
	DynP	-1.4420E+01	6.5500E+08	1.3700E+09	**16/50**	-1.2000E+01	-6.3400E+00	-3.0000E+00	0/50	-1.2448E+01	-3.1690E+00	0.0000E+00	0/50
	BP	-1.4412E+01	**-1.4026E+1**	**-1.3312E+1**	0/50	-1.1000E+01	-6.1400E+00	-3.0000E+00	0/50	-1.3241E+01	-2.9909E+00	0.0000E+00	0/50
	DF	-1.2409E+01	-8.9496E+00	-2.5059E+00	**2/50**	-6.8822E+00	-4.4224E+00	-1.0000E+00	0/50	-7.5419E+00	-2.5373E+00	0.0000E+00	0/50
	MDF	-1.3381E+01	-9.4978E+00	-5.5570E+00		-7.7028E+00	-4.6935E+00	-3.0000E+00	0/50	-9.0000E+00	-2.6924E+00	-8.5879E-01	0/50
		HHO				GWO				SCA			
	DP	-1.4012E+01	-6.4731E+00	-3.0000E+00	0/50	-8.6843E+00	-5.9645E+00	-3.4040E+00	0/50	-8.6758E+00	-5.9935E+00	-4.9857E+00	0/50
	SP	**-1.4847E+1**	-7.0321E+00	-3.0000E+00	0/50	-1.0729E+01	-6.7477E+00	-3.1056E+00	0/50	-8.9172E+00	-6.1569E+00	-5.0000E+00	0/50
	DynP	-1.4131E+01	-7.0369E+00	-3.0000E+00	0/50	-1.0323E+01	-6.3327E+00	-3.4136E+00	0/50	-8.8140E+00	-6.2034E+00	-5.0000E+00	0/50
	BP	-1.4667E+01	-7.1518E+00	-4.0000E+00	0/50	-1.0693E+01	-6.5900E+00	-3.4016E+00	0/50	-8.9323E+00	-6.1506E+00	-5.0000E+00	0/50
	DF	-8.2558E+00	-6.0613E+00	-4.1571E+00	0/50	-4.1582E+00	-3.4023E+00	-1.9893E+00	0/50	-6.9210E+00	-1.2754E+00	-8.5300E-01	0/50
	MDF	-1.1511E+01	-8.2842E+00	-5.4074E+00	0/50	-4.9112E+00	-3.1011E+00	-1.9968E+00	0/50	-7.2512E+00	-2.6052E+00	-1.8054E+00	0/50
F_4		GA				PSO				WOA			
	DP	7.6600E+03	7.2700E+07	4.5500E+08	**8/50**	7.0613E+03	8.0981E+03	1.7330E+04	0/50	1.0483E+04	1.6290E+04	2.8592E+04	0/50
	SP	7.3884E+03	1.0060E+04	1.6613E+04	0/50	7.0785E+03	8.1572E+03	1.3645E+04	0/50	8.6713E+03	1.3961E+04	2.8323E+04	0/50
	DynP	7.4400E+03	4.7500E+06	2.5800E+07	**26/50**	**7.0540E+3**	8.0975E+03	1.5373E+04	0/50	8.2571E+03	1.4527E+04	2.6237E+04	0/50
	BP	7.4803E+03	9.9773E+03	1.6033E+04	0/50	7.1269E+03	8.4330E+03	1.3684E+04	0/50	9.7355E+03	1.4895E+04	2.4232E+04	0/50
	DF	7.6049E+03	9.6407E+03	2.1833E+04	0/50	7.1732E+03	8.7407E+03	1.2864E+04	0/50	7.6406E+03	1.4718E+04	2.3793E+04	0/50
	MDF	7.2021E+03	1.0178E+04	1.5271E+04	0/50	7.1132E+03	**7.9120E+3**	1.0035E+04	0/50	8.7402E+03	1.2858E+04	1.7959E+04	0/50
		HHO				GWO				SCA			
	DP	8.7234E+03	9.8760E+03	1.7820E+04	0/50	1.0661E+04	3.6400E+08	4.5500E+08	**40/50**	9.0820E+03	1.2114E+04	2.5634E+04	0/50
	SP	8.3623E+03	9.2389E+03	1.0182E+04	0/50	9.1256E+03	1.4897E+04	2.7366E+04	0/50	8.9387E+03	1.0552E+04	1.3140E+04	0/50
	DynP	8.1216E+03	9.2254E+03	1.0629E+04	0/50	8.1461E+03	3.1200E+06	6.4200E+07	**13/50**	8.8897E+03	1.0689E+04	1.3255E+04	0/50
	BP	8.2157E+03	9.2196E+03	**9.9112E+3**	0/50	9.2046E+03	1.1000E+08	5.4800E+09	0/50	8.6830E+03	1.0750E+04	1.2174E+04	0/50
	DF	8.0593E+03	9.3261E+03	1.9760E+04	0/50	8.1043E+03	1.4628E+04	2.4610E+04	**2/50**	9.0726E+03	1.2481E+04	2.2662E+04	0/50
	MDF	7.9757E+03	9.1440E+03	1.0127E+04	0/50	8.1315E+03	1.0674E+04	1.5788E+04	0/50	8.2245E+03	1.1022E+04	1.1277E+04	0/50
F_5		GA				PSO				WOA			
	DP	6.8116E+02	6.8861E+02	7.2742E+02	0/50	**6.8063E+2**	**6.8068E+2**	**6.8083E+2**	0/50	6.9037E+02	7.2499E+02	8.7675E+02	0/50
	SP	6.8080E+02	6.8419E+02	6.9488E+02	0/50	6.8064E+02	**6.8068E+2**	6.8084E+02	0/50	6.8901E+02	7.0953E+02	8.5791E+02	0/50
	DynP	6.8092E+02	6.8449E+02	7.0622E+02	0/50	6.8064E+02	6.8070E+02	6.8088E+02	0/50	6.8687E+02	7.2056E+02	8.9160E+02	0/50
	BP	6.8159E+02	6.8752E+02	7.2097E+02	0/50	**6.8063E+2**	6.8069E+02	6.8092E+02	0/50	6.9306E+02	7.2194E+02	8.6388E+02	0/50
	DF	6.8494E+02	7.0223E+02	7.9190E+02	0/50	6.8065E+02	6.8092E+02	6.8264E+02	0/50	6.8916E+02	7.5257E+02	8.4048E+02	0/50
	MDF	6.8428E+02	6.9802E+02	7.4546E+02	0/50	6.8065E+02	6.8091E+02	6.8187E+02	0/50	6.9674E+02	7.4125E+02	8.4047E+02	0/50
		HHO				GWO				SCA			
	DP	6.8122E+02	6.8399E+02	6.8891E+02	0/50	6.8463E+02	7.1543E+02	8.9363E+02	0/50	6.8803E+02	6.9871E+02	7.2497E+02	0/50
	SP	6.8178E+02	6.8466E+02	6.8806E+02	0/50	6.8472E+02	7.1149E+02	8.6455E+02	0/50	6.8619E+02	6.9983E+02	7.2294E+02	0/50
	DynP	6.8187E+02	6.8439E+02	6.8853E+02	0/50	6.8353E+02	7.0731E+02	8.6402E+02	0/50	6.8626E+02	6.9827E+02	7.1797E+02	0/50
	BP	6.8205E+02	6.8470E+02	6.8866E+02	0/50	6.8494E+02	7.1467E+02	8.6327E+02	0/50	6.8641E+02	6.9807E+02	7.2230E+02	0/50
	DF	6.8172E+02	6.8494E+02	6.8806E+02	0/50	6.8325E+02	6.9788E+02	7.2981E+02	0/50	6.8276E+02	6.8614E+02	6.9155E+02	0/50
	MDF	6.8098E+02	6.8259E+02	6.8504E+02	0/50	6.8324E+02	6.9668E+02	7.2166E+02	0/50	6.8145E+02	6.8252E+02	6.8501E+02	0/50
F_6		GA				PSO				WOA			
	DP	-3.0416E+04	-2.9930E+04	-2.9495E+04	0/50	**-3.0667E+4**	**-3.0667E+4**	**-3.0667E+4**	0/50	-3.0489E+04	-3.0025E+04	-2.9359E+04	0/50
	SP	-3.0310E+04	-2.9992E+04	-2.9576E+04	0/50	-3.0666E+04	-3.0666E+04	-3.0666E+04	0/50	-3.0512E+04	-2.9967E+04	-2.9344E+04	0/50
	DynP	-3.0461E+04	-3.0083E+04	-2.9446E+04	0/50	-3.0666E+04	-3.0666E+04	-3.0666E+04	0/50	-3.0640E+04	-3.0043E+04	-2.9552E+04	0/50
	BP	-3.0339E+04	-3.0004E+04	-2.9346E+04	0/50	-3.0666E+04	-3.0666E+04	-3.0666E+04	0/50	-3.0499E+04	-2.9980E+04	-2.9438E+04	0/50
	DF	-3.0410E+04	-3.0016E+04	-2.9462E+04	0/50	-3.0665E+04	-3.0650E+04	-3.0500E+04	0/50	-3.0085E+04	-2.9433E+04	-2.8521E+04	0/50
	MDF	-3.0319E+04	-2.9954E+04	-2.9566E+04	0/50	-3.0665E+04	-3.0655E+04	-3.0512E+04	0/50	-3.0125E+04	-2.9597E+04	-2.8650E+04	0/50
		HHO				GWO				SCA			
	DP	-3.0660E+04	-3.0624E+04	-3.0592E+04	0/50	-3.0640E+04	-3.0433E+04	-3.0129E+04	0/50	-3.0665E+04	-3.0527E+04	-3.0348E+04	0/50
	SP	-3.0659E+04	-3.0624E+04	-3.0569E+04	0/50	-3.0649E+04	-3.0459E+04	-3.0168E+04	0/50	-3.0653E+04	-3.0516E+04	-3.0364E+04	0/50
	DynP	-3.0657E+04	-3.0624E+04	-3.0584E+04	0/50	-3.0655E+04	-3.0451E+04	-2.9535E+04	0/50	-3.0624E+04	-3.0519E+04	-3.0370E+04	0/50
	BP	-3.0653E+04	-3.0621E+04	-3.0550E+04	0/50	-3.0648E+04	-3.0467E+04	-2.9951E+04	0/50	-3.0653E+04	-3.0517E+04	-3.0343E+04	0/50
	DF	-3.0603E+04	-3.0441E+04	-3.0256E+04	0/50	-3.0499E+04	-3.0255E+04	-3.0019E+04	0/50	-3.0260E+04	-3.0402E+04	-2.9960E+04	0/50
	MDF	-3.0607E+04	-3.0458E+04	-3.0315E+04	0/50	-3.0466E+04	-3.0247E+04	-3.0067E+04	0/50	-3.0540E+04	-3.0453E+04	-3.0202E+04	0/50

Table 4. $F_7 - F_{11}$ Constrained benchmark problem optimizations min, median, max and violation/rerun results.

		Min	Median	Max	Violation /Rerun	Min	Median	Max	Violation /Rerun	Min	Median	Max	Violation /Rerun
F_7		GA				PSO				WOA			
	DP	7.6900E+08	2.0200E+09	2.3100E+09	**50/50**	7.6900E+08	1.4900E+09	1.5400E+09	**50/50**	7.6900E+08	1.4900E+09	2.3100E+09	**50/50**
	SP	7.4253E-01	4.3443E+01	1.4288E+03	0/50	1.4022E-01	1.1391E+00	1.0762E+01	0/50	7.4685E+01	2.7711E+03	9.0037E+03	0/50
	DynP	1.5157E-01	3.6227E+00	1.1055E+02	0/50	5.5073E-01	1.0366E+00	4.9602E+00	0/50	6.5187E-01	2.5794E+02	2.0018E+03	0/50
	BP	2.3300E+01	3.9500E+06	1.4500E+08	0/50	3.5630E-01	9.6739E-01	4.6260E+00	0/50	2.5300E+05	1.1000E+08	6.5400E+08	0/50
	DF	**3.4827E-3**	9.7869E-03	8.3462E-02	**2/50**	1.4624E-01	8.2791E-01	2.0351E+00	0/50	5.8042E-02	1.8137E+00	1.5750E+01	0/50
	MDF	3.6216E-03	**8.4220E-3**	**5.6001E-2**	0/50	7.3552E-02	5.5931E-01	1.1540E+01	0/50	2.1470E-01	9.9405E-01	2.5366E+00	0/50
		HHO				GWO				SCA			
	DP	7.6900E+08	1.4500E+09	1.5400E+09	**50/50**	1.5400E+09	1.5400E+09	1.5400E+09	**50/50**	7.6900E+08	1.5200E+09	1.5400E+09	**50/50**
	SP	2.2080E+00	2.5639E+03	9.8531E+03	0/50	1.2370E+05	1.9333E+06	1.0049E+07	**50/50**	1.5166E+05	1.1532E+06	3.4722E+06	**50/50**
	DynP	5.3317E-01	8.9800E+06	5.8800E+07	**25/50**	1.0264E+05	1.4500E+07	9.5600E+07	**50/50**	3.6228E+04	1.4800E+06	1.4400E+07	**50/50**
	BP	6.0400E+05	8.1900E+09	7.9400E+10	0/50	7.2900E+16	1.6100E+20	1.0000E+21	**50/50**	2.3200E+17	1.6600E+19	1.1500E+20	**50/50**
	DF	6.6059E-02	9.6061E-01	1.7436E+00	0/50	4.9030E-01	1.2273E+00	1.4370E+01	**48/50**	4.7633E-01	1.8599E+00	1.5612E+01	**37/50**
	MDF	6.0100E-02	9.5548E-01	1.6574E+00	0/50	4.1343E-01	9.8974E-01	2.0846E+00	**34/50**	4.5584E-01	1.1850E+00	7.7885E+00	**30/50**
F_8		GA				PSO				WOA			
	DP	2.8100E+01	4.2900E+07	7.1400E+08	**5/50**	2.4761E+01	**3.0135E+1**	1.7247E+02	0/50	3.1923E+01	8.7474E+01	7.4338E+02	0/50
	SP	2.7841E+01	3.4923E+01	6.5645E+01	0/50	2.4554E+01	5.1452E+01	5.1536E+02	0/50	3.1119E+01	8.0436E+01	5.1029E+02	0/50
	DynP	2.8466E+01	3.4777E+01	4.7065E+01	0/50	**2.4514E+1**	1.3329E+02	3.0929E+03	0/50	2.7380E+01	7.3606E+01	6.6759E+02	0/50
	BP	2.7109E+01	3.6073E+01	1.2774E+02	0/50	2.4934E+01	6.2449E+01	5.1281E+02	0/50	2.7849E+01	1.1773E+02	1.5104E+03	0/50
	DF	2.8580E+01	5.0897E+01	2.6837E+02	0/50	2.5601E+01	4.2089E+02	5.7054E+03	0/50	6.4577E+01	4.9207E+02	2.2627E+03	0/50
	MDF	2.8438E+01	5.7700E+01	1.6308E+02	0/50	2.4807E+01	3.0882E+01	4.6733E+01	0/50	3.7013E+01	2.8785E+02	1.3085E+03	0/50
		HHO				GWO				SCA			
	DP	3.1426E+01	3.9847E+01	7.7474E+01	0/50	3.8061E+01	2.2100E+98	7.1400E+98	**20/50**	6.6254E+01	2.4745E+02	2.2109E+03	0/50
	SP	2.6201E+01	3.6527E+01	5.1620E+01	0/50	4.0130E+01	5.2279E+02	1.0920E+03	0/50	5.0865E+01	1.3329E+02	8.7627E+02	0/50
	DynP	2.9480E+01	3.8372E+01	5.0927E+01	0/50	4.3360E+01	4.8022E+02	1.1820E+03	0/50	5.3630E+01	1.9465E+02	9.1259E+02	0/50
	BP	2.8947E+01	3.9180E+01	5.7541E+01	0/50	3.7436E+01	5.1109E+02	1.1575E+03	0/50	4.5312E+01	1.6345E+02	8.6435E+02	0/50
	DF	3.1432E+01	3.6203E+01	5.5180E+01	0/50	3.5598E+01	3.8116E+02	5.6209E+03	0/50	2.9768E+01	3.9755E+01	4.5553E+01	0/50
	MDF	2.8712E+01	3.5210E+01	5.2556E+01	0/50	3.8744E+01	2.5088E+02	1.0172E+03	0/50	3.0214E+01	3.8074E+01	**4.0712E+1**	0/50
F_9		GA				PSO				WOA			
	DP	-3.4000E-01	1.8000E+09	1.0000E+10	**18/50**	-8.5850E-01	-5.1049E-01	-8.7110E-04	0/50	-5.3284E-02	-2.5691E-03	0.0000E+00	0/50
	SP	-7.5645E-01	-4.2648E-01	-2.5016E-02	0/50	-7.9316E-02	9.9843E-02	4.7204E+00	0/50	0.0000E+00	1.4821E-03	3.0169E-02	0/50
	DynP	-8.5381E-01	**-5.3578E-1**	**-8.7424E-1**	0/50	-6.2445E-01	-2.9489E-02	9.8800E-08	0/50	-1.0944E-01	-5.1353E-03	1.1900E-06	0/50
	BP	-8.5205E-01	1.6108E+06	5.3899E+07	0/50	-5.3016E-02	3.2100E+05	1.2500E+07	0/50	0.0000E+00	5.3373E+03	1.1773E+05	0/50
	DF	1.3808E-04	4.2931E+02	1.3800E+03	**50/50**	-1.3790E-01	-5.0149E-03	0.0000E+00	0/50	-1.3123E-02	-8.0271E-04	0.0000E+00	0/50
	MDF	2.5898E-02	3.1561E+02	1.2777E+04	**50/50**	-6.9450E-01	-1.1121E-01	0.0000E+00	0/50	-6.8491E-01	-2.2246E-01	-1.0640E-04	0/50
		HHO				GWO				SCA			
	DP	-4.9349E-01	-5.3319E-02	0.0000E+00	0/50	-2.1989E-01	-1.5071E-02	0.0000E+00	0/50	-2.1557E-02	-5.0714E-04	0.0000E+00	0/50
	SP	**-8.8537E-1**	-2.4485E-02	1.1100E-09	0/50	5.9632E-01	1.1580E+02	3.7148E+02	0/50	0.0000E+00	0.0000E+00	0.0000E+00	0/50
	DynP	-2.4888E-01	-1.6252E-02	0.0000E+00	0/50	-2.4042E-01	8.9500E-01	1.1000E+01	0/50	0.0000E+00	0.0000E+00	0.0000E+00	0/50
	BP	-2.8517E-01	-1.6071E-02	0.0000E+00	0/50	1.8700E+02	2.8600E+07	2.3200E+08	0/50	0.0000E+00	0.0000E+00	0.0000E+00	0/50
	DF	-5.3683E-01	-3.2081E-02	0.0000E+00	0/50	-1.3848E-01	-2.2046E-02	0.0000E+00	0/50	0.0000E+00	0.0000E+00	0.0000E+00	0/50
	MDF	-6.3057E-01	-1.9026E-01	0.0000E+00	0/50	-4.9176E-01	-1.9637E-01	-1.1593E-03	0/50	-2.2459E-01	-5.8725E-03	0.0000E+00	0/50
F_{10}		GA				PSO				WOA			
	DP	-6.2700E+03	8.0000E+08	1.6700E+09	**24/50**	-6.9637E+03	3.3300E+07	1.6700E+09	**1/50**	-6.9512E+03	-6.7141E+03	-2.2589E+03	0/50
	SP	-6.2299E+03	-4.2141E+03	-1.2070E+03	0/50	-6.9618E+03	**-6.9617E+3**	-6.9612E+03	0/50	-6.9594E+03	-6.5604E+03	-1.3584E+03	0/50
	DynP	-6.2327E+03	-4.1349E+03	-1.4343E+03	0/50	-6.9619E+03	**-6.9617E+3**	-6.9599E+03	0/50	-6.9582E+03	-6.8107E+03	-3.5622E+03	0/50
	BP	-6.3350E+03	-3.9469E+03	-1.2165E+03	0/50	-6.9618E+03	**-6.9617E+3**	**-6.9614E+3**	0/50	-6.9616E+03	-6.5639E+03	-1.3983E+03	0/50
	DF	-6.7155E+03	-3.9005E+03	-1.4346E+03	0/50	-6.9618E+03	-6.9352E+03	-5.6506E+03	0/50	-6.9618E+03	-6.0734E+03	-1.3373E+03	0/50
	MDF	-6.6866E+03	-3.4820E+03	-1.2070E+03	0/50	-6.9618E+03	-6.9615E+03	-6.9592E+03	0/50	-6.9618E+03	-5.5840E+03	-1.3425E+03	0/50
		HHO				GWO				SCA			
	DP	**-6.9641E+3**	-6.9587E+03	-6.9039E+03	0/50	-6.9596E+03	-6.8404E+03	-3.3150E+03	0/50	-6.8926E+03	-6.4785E+03	-2.5500E+03	0/50
	SP	-6.9618E+03	-6.9611E+03	-6.9537E+03	0/50	-6.9517E+03	-6.9187E+03	-6.8773E+03	0/50	-6.9259E+03	-6.7343E+03	-6.3210E+03	0/50
	DynP	-6.9618E+03	-6.9602E+03	-6.8902E+03	0/50	-6.9596E+03	-6.9158E+03	-6.8470E+03	0/50	-6.9326E+03	-6.7412E+03	-6.4378E+03	0/50
	BP	-6.9618E+03	-6.9616E+03	-6.9595E+03	0/50	-6.9576E+03	-6.9222E+03	-6.8620E+03	0/50	-6.9403E+03	-6.7671E+03	-6.4699E+03	0/50
	DF	-6.9618E+03	-6.9595E+03	-6.9302E+03	0/50	-6.9406E+03	-6.6952E+03	-6.1748E+03	0/50	-6.9554E+03	-6.8528E+03	-6.6157E+03	0/50
	MDF	-6.9618E+03	-6.9600E+03	-6.9502E+03	0/50	-6.9402E+03	-6.7552E+03	-6.4356E+03	0/50	-6.9506E+03	-6.8512E+03	-6.6185E+03	0/50
F_{11}		GA				PSO				WOA			
	DP	7.4912E-01	8.4620E-01	9.9556E-01	0/50	**7.4900E-1**	**7.4902E-1**	**7.4915E-1**	0/50	**7.4900E-1**	7.5734E-01	9.9900E-01	0/50
	SP	9.9973E-01	9.4036E-01	8.0224E-01	0/50	7.8024E-01	9.4848E-01	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50
	DynP	9.9684E-01	9.1104E-01	7.5140E-01	0/50	7.5038E-01	9.0091E-01	1.0000E+00	0/50	9.9606E-01	9.9987E-01	1.0000E+00	0/50
	BP	9.9939E-01	9.2046E-01	8.3013E-01	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50
	DF	9.9939E-01	9.2046E-01	7.5013E-01	0/50	7.9356E-01	9.5823E-01	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50
	MDF	7.5020E-01	7.5355E-01	7.7755E-01	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50
		HHO				GWO				SCA			
	DP	**7.4900E-1**	7.4918E-01	7.5191E-01	0/50	7.4902E-01	7.4958E-01	7.6021E-01	0/50	7.4911E-01	7.4978E-01	7.5143E-01	0/50
	SP	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	3.2411E+00	5.2038E+01	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50
	DynP	7.5661E-01	9.1345E-01	1.0000E+00	0/50	7.8537E-01	9.5531E-01	1.1205E+00	0/50	7.9099E-01	9.8554E-01	1.0000E+00	0/50
	BP	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	3.6225E+07	6.9166E+08	**12/50**	1.0000E+00	1.0000E+00	1.0000E+00	0/50
	DF	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50
	MDF	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50	1.0000E+00	1.0000E+00	1.0000E+00	0/50

Table 5. Among the constraint handling methods using metaheuristic algorithms, the best minimum, median and maximum case numbers and violations obtained from Tables 3 and 4.

The best case numbers for minimum, median, maximum and violation numbers amoung all constraint handling methods using metaheuristic algorithms												
	GA				PSO				WOA			
	Minimum	Median	Maximum	Violations	Minimum	Median	Maximum	Violations	Minimum	Median	Maximum	Violations
DP	0	0	0	**105/550**	**5**	**6**	**5**	**51/550**	2	0	0	**50/550**
SP	0	0	0	0/550	0	2	0	0/550	0	0	0	0/550
DynP	0	1	1	**50/550**	2	1	0	0/550	0	0	0	0/550
BP	0	1	1	0/550	1	1	1	0/550	0	0	0	0/550
DF	2	0	0	**52/550**	0	0	0	0/550	0	0	0	0/550
MDF	1	1	1	**56/550**	0	1	0	0/550	0	0	0	0/550
	HHO				GWO				SCA			
	Min	Median	Max	Violations	Min	Median	Max	Violations	Min	Median	Max	Violations
DP	3	0	0	**51/550**	0	0	0	**110/550**	1	0	0	**50/550**
SP	2	0	0	0/550	0	0	0	**50/550**	0	0	0	**50/550**
DynP	0	0	0	**25/550**	0	0	0	**63/550**	0	0	0	**50/550**
BF	0	0	1	0/550	0	0	0	**62/550**	0	0	0	**50/550**
DF	0	0	0	0/550	0	0	0	**58/550**	0	0	0	**37/550**
MDF	0	0	0	0	0	0	0	**34/550**	0	0	0	**30/550**
The best case numbers for minimum, median, maximum and violation numbers between DF and MDF constraint handling methods using metaheuristic algorithms												
	GA				PSO				WOA			
	Min	Median	Max	Violations	Min	Median	Max	Violations	Min	Median	Max	Violations
DF	2	0	1	**52/550**	4	1	1	0/550	1	0	0	0/550
MDF	2	2	1	**56/550**	**7**	**7**	**6**	0/550	1	1	0	0/550
	HHO				GWO				SCA			
	Min	Median	Max	Violations	Min	Median	Max	Violations	Min	Median	Max	Violations
DF	1	0	0	0/550	0	0	0	**58/550**	0	0	0	**37/550**
MDF	2	1	1	0/550	0	0	1	**34/550**	0	0	1	**30/550**

Table 6. Significance values using Wilcoxon test of $F_1 - F_{11}$ benchmark problems for all metaheuristic algorithms according to MDF method and the other constraint handling methods.

	GA					PSO					WOA				
	DP	SP	DynP	BP	MF	DP	SP	DynP	BP	MF	DP	SP	DynP	BP	MF
F_1	7.6E-10	2.5E-02	2.3E-03	3.7E-05	1.0E-06	6.1E-10	1.2E-03	7.3E-10	4.4E-07	2.3E-07	6.8E-10	8.0E-06	2.8E-03	1.0E-06	1.7E-09
F_2	8.5E-10	4.3E-04	5.2E-02	2.0E-06	4.8E-07	2.0E-03	**7.5E-1**	4.4E-01	**7.9E-1**	1.6E-05	6.1E-10	5.6E-10	6.1E-10	6.1E-10	1.0E-06
F_3	7.6E-10	7.6E-10	**6.8E-1**	7.6E-10	1.1E-09	7.6E-10	7.6E-10	7.6E-10	7.6E-10	2.7E-05	5.8E-10	5.8E-10	6.1E-10	5.8E-10	5.7E-08
F_4	2.5E-07	1.3E-02	2.4E-04	4.8E-04	1.6E-08	7.8E-03	1.0E-06	1.1E-02	1.1E-09	8.5E-10	6.1E-10	4.2E-09	7.7E-10	7.7E-10	1.7E-09
F_5	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.4E-04	7.6E-10	7.6E-10	7.6E-10	7.6E-10	3.4E-01	1.0E-06	8.2E-10	1.4E-08	3.7E-07	7.6E-09
F_6	3.5E-05	1.0E-06	1.8E-09	3.5E-08	1.1E-07	7.6E-10	7.6E-10	7.6E-10	7.6E-10	5.4E-03	6.1E-10	5.8E-10	5.8E-10	6.5E-10	7.0E-09
F_7	7.6E-10	7.5E-10	3.0E-06	2.2E-10	1.7E-10	6.2E-10	6.0E-10	6.2E-10	7.5E-10	7.4E-10	5.6E-10	5.6E-10	5.6E-10	5.6E-10	4.4E-10
F_8	6.0E-09	1.1E-08	7.6E-10	**4.5E-1**	7.2E-02	8.5E-10	1.7E-09	7.6E-10	7.6E-10	1.4E-08	5.2E-04	1.0E-06	3.0E-06	6.5E-10	6.1E-10
F_9	7.6E-10	7.6E-10	2.0E-03	2.2E-09	7.6E-10	8.0E-10	3.5E-08	7.6E-10	7.6E-10	7.6E-10	5.5E-10	1.3E-03	5.9E-10	5.1E-02	7.1E-10
F_{10}	9.5E-09	9.1E-10	7.6E-10	5.7E-09	4.0E-06	2.8E-09	2.7E-09	2.8E-09	2.8E-09	2.8E-09	5.0E-03	6.0E-09	9.5E-03	2.4E-03	2.0E-04
F_{11}	7.6E-10	1.2E-09	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.2E-09	7.6E-10	7.6E-10	7.6E-10	5.6E-10	5.6E-10	5.6E-10	5.6E-10	5.6E-10
	HHO					GWA					SCA				
	DP	SP	DynP	BP	MF	DP	SP	DynP	BP	MF	DP	SP	DynP	BP	MF
F_1	7.6E-10	7.6E-10	7.3E-02	1.4E-08	1.2E-03	1.8E-02	7.6E-10	7.2E-04	7.6E-10	1.7E-02	9.1E-10	7.6E-10	7.6E-10	7.6E-10	4.1E-08
F_2	7.6E-10	4.0E-09	7.6E-10	1.0E-09	3.5E-03	7.6E-10	1.2E-09	9.1E-10	1.2E-09	1.4E-09	7.6E-10	7.6E-10	7.6E-10	7.6E-10	9.4E-02
F_3	1.5E-09	2.5E-04	1.6E-05	4.7E-01	1.7E-01	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.2E-09	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10
F_4	7.6E-10	7.6E-10	4.3E-09	7.6E-10	3.4E-09	1.9E-05	2.5E-08	2.5E-08	2.9E-08	1.1E-08	7.6E-10	7.6E-10	7.6E-10	7.6E-10	4.8E-07
F_5	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.0E-09	3.3E-08	7.6E-10	5.2E-04	7.6E-10	1.1E-03	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.4E-02
F_6	1.0E-02	2.6E-01	3.5E-01	1.7E-01	3.6E-09	7.6E-10	7.6E-10	7.6E-10	7.6E-10	3.6E-02	7.6E-10	7.6E-10	7.6E-10	7.6E-10	5.9E-07
F_7	7.6E-10	7.6E-10	8.0E-10	7.5E-10	7.4E-10	7.6E-10	7.6E-10	8.0E-10	5.3E-10	7.3E-10	7.6E-10	7.6E-10	8.0E-10	2.3E-10	3.8E-10
F_8	7.6E-10	5.0E-06	1.9E-01	4.6E-03	1.0E-09	8.3E-03	3.7E-03	9.6E-02	8.4E-01	8.1E-01	7.6E-10	1.3E-07	3.7E-03	7.6E-10	7.6E-10
F_9	1.4E-07	9.7E-07	3.6E-08	3.8E-02	1.1E-09	2.9E-07	4.0E-06	9.1E-10	1.4E-08	1.4E-08	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10
F_{10}	**7.8E-1**	**7.3E-1**	**8.6E-1**	2.4E-01	2.7E-01	2.9E-07	4.0E-06	9.1E-10	1.4E-08	1.4E-08	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10
F_{11}	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10	7.6E-10

Table 7. Average processing time (s) of $F_1 - F_{11}$ benchmark problems.

	F_1						F_2						F_2					
	GA	PSO	WOA	HHO	GWO	SCA	GA	PSO	WOA	HHO	GWO	SCA	GA	PSO	WOA	HHO	GWO	SCA
DP	45.64	2.71	0.55	1.77	0.61	0.56	93.09	22.32	19.93	51.74	22.80	21.92	143.16	2.94	0.79	2.21	0.95	1.10
SP	29.58	2.68	0.34	0.99	0.39	0.34	38.09	14.96	11.43	11.37	12.99	13.05	29.82	2.79	0.46	1.44	0.62	0.67
DynP	29.38	2.41	0.40	1.14	0.48	0.58	37.76	14.48	11.59	12.11	11.95	12.93	29.96	2.83	0.55	1.58	0.77	0.59
BP	47.01	2.81	0.43	1.19	0.57	0.47	58.89	16.96	11.58	12.27	12.96	12.69	49.39	3.40	0.58	1.85	0.71	0.62
DF	29.52	4.58	1.27	1.66	1.37	1.44	37.70	89.75	49.50	62.38	56.88	55.09	30.39	5.96	1.67	2.51	1.93	1.75
MDF	50.74	5.15	1.18	1.64	1.40	1.16	72.84	95.98	51.74	67.29	59.85	60.17	52.38	6.05	1.73	2.29	1.95	1.80
	F_4						F_5						F_6					
DP	82.09	2.75	0.59	1.79	0.72	0.74	47.77	2.74	0.65	1.85	0.88	0.83	48.62	2.73	0.60	1.79	0.73	0.72
SP	29.73	2.52	0.36	1.05	0.48	0.51	29.53	2.48	0.40	1.18	0.49	0.55	29.41	2.98	0.36	1.14	0.43	0.50
DynP	30.51	2.62	0.43	1.18	0.64	0.46	29.48	2.60	0.45	1.26	0.55	0.48	29.42	2.52	0.43	1.26	0.62	0.44
BP	48.12	2.77	0.49	1.29	0.55	0.48	47.82	3.01	0.50	1.29	0.56	0.49	48.06	2.92	0.46	1.32	0.53	0.44
DF	29.96	5.11	1.24	1.65	1.37	1.27	29.87	5.29	1.36	1.70	1.47	1.40	29.85	4.92	1.22	1.56	1.40	1.20
MDF	49.81	5.98	1.47	1.67	1.54	1.41	48.74	5.95	1.57	1.87	1.53	1.45	48.74	4.59	1.42	1.67	1.39	1.30
	F_7						F_8						F_9					
DP	121.63	2.91	0.63	1.91	0.69	0.64	77.13	2.82	0.62	1.75	0.74	0.68	95.98	2.61	0.48	1.49	0.56	0.55
SP	29.05	2.46	0.34	1.00	0.38	0.37	29.69	2.60	0.37	1.00	0.52	0.44	29.37	2.49	0.34	0.97	0.43	0.40
DynP	29.43	2.44	0.35	0.99	0.43	0.40	29.85	3.01	0.46	1.18	0.63	0.52	29.56	2.76	0.41	1.13	0.64	0.46
BP	47.90	2.74	0.40	1.17	0.45	0.46	48.24	2.97	0.49	1.46	0.56	0.55	49.38	2.94	0.52	1.64	0.59	0.59
DF	30.02	4.78	1.04	1.70	1.13	1.13	30.08	4.98	1.25	2.11	1.45	1.33	30.43	4.74	1.19	2.22	1.34	1.22
MDF	52.97	4.83	1.47	1.76	1.56	1.43	50.75	4.88	1.44	1.83	1.54	1.37	53.21	5.06	1.42	1.70	1.77	1.55
	F_{10}						F_{11}						Total of processing time from F_1 to F_{11}					
DP	97.15	2.83	0.57	1.78	0.58	0.59	138.82	2.75	0.44	1.38	0.44	0.43	991.09	50.13	25.86	69.46	29.70	28.75
SP	29.50	2.41	0.36	1.07	0.38	0.35	29.15	2.37	0.31	1.05	0.31	0.33	332.93	40.73	15.05	22.26	17.41	17.50
DynP	29.67	2.62	0.43	1.29	0.48	0.43	29.05	2.53	0.38	1.17	0.38	0.42	334.06	40.82	15.88	24.30	17.56	17.71
BP	49.63	2.54	0.46	1.71	0.45	0.45	54.75	2.60	0.47	1.77	0.52	0.47	549.18	45.66	16.37	26.96	18.44	17.71
DF	29.72	5.10	1.23	1.87	1.32	1.27	54.75	2.60	0.47	1.77	0.52	0.47	362.28	137.79	61.44	81.14	70.18	67.58
MDF	50.76	4.84	1.43	1.75	1.53	1.30	51.03	4.49	1.16	1.65	1.29	1.25	581.98	147.80	66.02	85.12	75.34	74.21

The statistical results in which are minimum, median, maximum, violation numbers and Wilcoxon tests were obtained with respect to each algorithm crosses to all constraint handling methods. Tables 3 and 4 present minimum, median and maximum values as well as violation numbers during reruns. Especially, in order to emphasize the best cases for minimum, maximum and median values obtained are dyed blue and typed in bold. Also, constraint violation numbers are dyed red and typed in bold. Specifically, violation numbers are given in form of a fraction. The numerator presents violation numbers and the denominator presents independent rerun numbers that are 50 for each one of them. In Table 5, as a result of the analysis of the constraint handling methods and metaheuristic algorithms used together, numbers of best cases for minimum, maximum and median, as well as the number of constraint violations, are given. In this way, the results in Tables 3 and 4 have been summarized in Table 5, making the achievements of algorithms and constraint handling methods more understandable according to the number of cases. Also, in Table 5, in order to emphasize the numbers of the best case for each minimum, maximum and median values obtained are dyed blue and typed in bold. Also, total constraint violation numbers are dyed red and typed in bold. After that, in order to illustrate significance values, Wilcoxon test results are presented in Table 6. In Table 6, if the significance value is higher than 0.05 then, it is dyed red and typed in bold. Subsequently, Table **??** presents the average processing time with regard to each algorithm and each constraint handling method.

In the upper part of Table 5, the results of the GA, PSO, WOA, HHO, GWO and SCA algorithms according to the DP, SP, DynP, DF and MDF constraint handling methods are examined, and the results obtained from the $F_1 - F_{11}$ constrained benchmark functions are summarized. As an example, PSO-DP, which gives the best case numbers in the table, is examined. While PSO-DP found 5 cases for the best minimum, 6 cases for the best median, and 5 cases for the best maximum. However, when the constraint violation was examined, 51 constraint violations have occurred among 550 trials in $F_1 - F_{11}$ functions. Comparisons of the results obtained by using other constraint handling methods such as SP, DynP, DP, DF and MDF using PSO can be made easily. Globally, The GWO-DP algorithm performed more constraint violations than other algorithms, that is, it has made 110 constraint violations in 550 trials. In the lower part of Table 5, the results of the GA, PSO, WOA, HHO, GWO and SCA algorithms according to DF and MDF constraint handling methods are examined, and the results obtained from the $F_1 - F_{11}$ constrained benchmark functions are summarized. As an example, PSO-MDF, which gives the best results in the table, is examined. While PSO-MDF found 7 cases for the best minimum, 7 cases for the best median and 7 cases for the best maximum. Also, no constraint violations occurred. The highest number of constraint violations occurred in the GWO-DF method.

4.2. Pressure bessel design

The pressure vessel design problem is a very common benchmark problem known and used by many researchers. In this problem, the objective function is to make the cost of this cylinder minimal with reference to its 4 different variables that must satisfy 4 constraints [8, 26].

In this study, this problem was tried to be solved with different metaheuristic algorithms and constraint handling methods. Their results of minimum, median, maximum, proceesing times, violations and statistical tests are given in detail in Table 8. While PSO-DP produced the best minimum value, SCA-MDF achieved the best median and maximum values. Also, there is no violation for this problem for all results and there are significant differences between MDF and others, that is, significance values are less than 0.05. Furthermore, the study and studies in the literature are comparatively given in Table 8. Most of these studies' results are obtained by using SP. When these results are compared, PSO-DP achieved better results than the other studies in Table 8.

Table 8. Pressure vessel design statistical results.

Minimum, median, maximum, violations and average processing time results for pressure vessel design											
DP						Dynp					
	Min	Median	Max	Violation /Rerun	Time (sec)	Min	Median	Max	Violation /Rerun	Time (sec)	
GA	5.211E+07	5.394E+07	5.451E+07	0/50	65.41	5.251E+07	5.399E+07	5.444E+07	0/50	29.96	
PSO	**5.875E+3**	6.233E+03	7.308E+03	0/50	3.13	5.884E+03	6.209E+03	7.318E+03	0/50	2.47	
WOA	5.936E+03	7.582E+03	1.077E+04	0/50	0.65	6.247E+03	7.794E+03	1.103E+04	0/50	0.42	
HHO	5.930E+03	6.685E+03	7.457E+03	0/50	1.80	5.940E+03	6.395E+03	7.521E+03	0/50	1.16	
GWO	6.071E+03	6.287E+03	7.643E+03	0/50	0.65	5.969E+03	6.678E+03	7.589E+03	0/50	0.44	
SCA	6.171E+03	6.679E+03	7.080E+03	0/50	0.69	6.039E+03	6.727E+03	8.372E+03	0/50	0.47	
SP						BP					
GA	5.333E+07	5.396E+07	5.444E+07	0/50	29.33	5.232E+07	5.395E+07	5.453E+07	0/50	52.70	
PSO	5.885E+03	6.193E+03	6.951E+03	0/50	2.54	5.885E+03	6.218E+03	7.026E+03	0/50	2.51	
WOA	6.054E+03	7.724E+03	1.086E+04	0/50	0.37	6.119E+03	7.495E+03	9.467E+03	0/50	0.48	
HHO	5.946E+03	6.492E+03	7.475E+03	0/50	0.96	5.943E+03	6.395E+03	7.606E+03	0/50	1.25	
GWO	5.981E+03	6.668E+03	7.560E+03	0/50	0.38	5.998E+03	6.777E+03	7.551E+03	0/50	0.55	
SCA	6.171E+03	6.764E+03	8.032E+03	0/50	0.37	6.120E+03	6.798E+03	8.153E+03	0/50	0.46	
MF						MDF					
GA	5.360E+07	5.417E+07	5.461E+07	0/50	29.77	5.344E+07	5.417E+07	5.456E+07	0/50	51.73	
PSO	5.885E+03	6.309E+03	7.319E+03	0/50	5.09	5.885E+03	6.291E+03	7.309E+03	0/50	5.46	
WOA	6.158E+03	2.289E+04	3.458E+05	0/50	1.22	5.935E+03	1.523E+04	1.182E+05	0/50	1.23	
HHO	5.960E+03	6.267E+03	7.898E+03	0/50	2.12	5.927E+03	6.226E+03	7.500E+03	0/50	1.67	
GWO	6.123E+03	6.761E+03	7.305E+03	0/50	1.36	5.944E+03	6.748E+03	7.196E+03	0/50	1.39	
SCA	6.022E+03	6.026E+03	6.449E+03	0/50	1.56	5.936E+03	**6.024E+3**	**6.310E+3**	0/50	1.51	
Significance value results with reference to Wilxocon Test for pressure vessel design											
	DP	SP	DynP	BP	MF		DP	SP	DynP	BP	MF
GA	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.4E-01	HHO	2.4E-04	2.2E-04	9.4E-05	1.3E-02	3.2E-02
PSO	1.7E-04	4.1E-08	2.0E-07	7.6E-10	4.5E-08	GWA	8.5E-04	3.1E-08	7.6E-10	1.2E-09	9.0E-02
WOA	6.7E-10	6.7E-10	7.7E-10	6.3E-10	2.6E-09	SCA	7.6E-10	7.6E-10	7.6E-10	7.6E-10	2.7E-04
Optimization results by different algorithm in literature for pressure vessel design											
GA [26]	CMVHHO [8]	GSA [8]	MVO [8]	PSO [8]	GPEA [9]	GA-DP	PSO-DP	WOA-MFD	HHO-MFD	GWO-MDF	SCA-MDF
6.059 E+03	6.061 E+03	8.538 E+03	6.040 E+03	6.061 E+03	6.059 E+03	5.211 E+07	**5.875 E+3**	5.935 E+03	5.927 E+03	5.944 E+03	5.936 E+03

4.3. Welded beam design

The welded beam design problem is a very common benchmark problem known by many researchers [22]. In this problem, the objective function has been generated such that making the production cost minimize with reference to its 4 different variables that must satisfy 5 constraints [22].
In this study, this problem was tried to be solved with different metaheuristic algorithms and constraint handling methods. Their results of minimum, median, maximum, processing times, violations and statistical tests are given in detail in Table 9. While PSO-DP produced the best minimum and median values, PSO-SP achieved the best maximum value. But, violations have occurred in GA-SP for this problem and also there are significant differences between MDF and others, that is, significance values are less than 0.05 except for PSO-MF. Furthermore, the study and studies in the literature are comparatively given in Table 8. Most of these results are obtained by using SP. When these results are compared, PSO-DP achieved better results than the other studies in Table 9.

4.4. Speed reducer design

In the speed reducer design problem, the objective function has been generated such that making the weight of the speed reducer minimize with reference to 4 different variables that must satisfy 11 constraints [25].
In this study, this problem was tried to be solved with different metaheuristic algorithms and constraint handling methods. Their results of minimum, median, maximum, processing times, violations and statistical tests are given in Table 10. In this problem, PSO-DP and HHO-DP achieved the best minimum, PSO-DP produced the best median value and the best maximum values are obtained by GA-SP and GA-DynP. There is merely one constraint violation that is occurred in GA-DP and also there are significant differences between MDF and others, that is, significance values are less than 0.05. Furthermore, the study and studies in the literature are comparatively given in Table 10. Most of these results are obtained by using SP. When these results are compared, PSO-DP achieved better results than the other studies in Table 10.

4.5. Active filter design

In the active filter design problem, the circuit is to design using standard components, that are specific and discrete values, with respect to the desired operating amplitude and frequency [10]. Therefore, the objective function has included the difference between the operating condition of the filter and the desired operating condition of the filter. While the objective function designed for the active filter is being minimized, 8 discrete component variables should satisfy 2 constraints [10].
In this study, this problem was tried to be solved with different metaheuristic algorithms and constraint handling methods. Their results of minimum, median, maximum, proceesing times, violations and statistical tests are given in Table 11. In this problem, PSO-MDF achieved best minimum and best median is obtained by HHO-MDF. Also, the best maximum value is obtained by PSO-SP. There is constraint violation that is occurred in GA-DP and also some significance values between MDF and others, such as GA-DP, GA-DynP, PSO-BP and GWA-DynP, are higher than 0.05 that means results are not significant difference. Main reason of this, variables are discrete. Furthermore, the study and studies in the literature are comparatively depicted in Table 11. When these results are compared, PSO-MDF achieved better results than the other studies in Table 11.

Table 9. Welded beam design statistical results.

Minimum, median, maximum, violations and average processing time results for welded beam design											
DP						DynP					
	Min	Median	Max	Violation /Rerun	Time (sec)	Min	Median	Max	Violation /Rerun	Time (sec)	
GA	5.926E+00	8.237E+00	9.918E+00	0/50	68.43	8.216E+00	9.460E+00	1.109E+01	0/50	29.58	
PSO	**1.588**	**1.588**	1.598E+00	0/50	3.24	1.589E+00	1.589E+00	1.592E+00	0/50	2.51	
WOA	1.628E+00	1.922E+00	3.642E+00	0/50	0.65	1.628E+00	2.001E+00	4.039E+00	0/50	0.44	
HHO	1.619E+00	1.704E+00	2.101E+00	0/50	1.81	1.599E+00	1.728E+00	2.164E+00	0/50	1.27	
GWO	1.597E+00	1.625E+00	1.651E+00	0/50	0.66	1.602E+00	1.621E+00	1.645E+00	0/50	0.51	
SCA	1.638E+00	1.702E+00	1.758E+00	0/50	0.58	1.623E+00	1.700E+00	1.777E+00	0/50	0.50	
SP						BP					
GA	1.291E+09	1.291E+09	1.291E+09	**50/50**	29.78	7.540E+00	9.128E+00	1.085E+01	0/50	54.92	
PSO	1.589E+00	1.589E+00	**1.589**	0/50	2.71	1.589E+00	1.589E+00	1.590E+00	0/50	2.66	
WOA	1.643E+00	2.010E+00	2.937E+00	0/50	0.38	1.630E+00	1.920E+00	3.224E+00	0/50	0.48	
HHO	1.600E+00	1.700E+00	1.933E+00	0/50	1.02	1.596E+00	1.735E+00	2.405E+00	0/50	1.35	
GWO	1.598E+00	1.624E+00	1.646E+00	0/50	0.43	1.603E+00	1.624E+00	1.649E+00	0/50	0.57	
SCA	1.631E+00	1.701E+00	1.783E+00	0/50	0.39	1.633E+00	1.705E+00	1.796E+00	0/50	0.48	
MF						MDF					
GA	8.889E+00	9.961E+00	1.130E+01	0/50	29.89	8.211E+00	9.221E+00	1.113E+01	0/50	51.36	
PSO	1.589E+00	1.594E+00	1.739E+00	0/50	5.38	1.589E+00	1.590E+00	1.606E+00	0/50	5.29	
WOA	1.727E+00	2.702E+00	4.191E+00	0/50	1.30	1.765E+00	2.696E+00	3.728E+00	0/50	1.31	
HHO	1.592E+00	1.600E+00	1.616E+00	0/50	2.37	1.592E+00	1.601E+00	1.630E+00	0/50	1.95	
GWO	1.599E+00	1.614E+00	1.630E+00	0/50	1.40	1.598E+00	1.614E+00	1.616E+00	0/50	1.46	
SCA	1.595E+00	2.163E+00	5.120E+00	0/50	1.55	1.591E+00	2.002E+00	3.423E+00	0/50	1.59	
Significance value results with reference to Wilxocon Test for welded beam design											
	DP	SP	DynP	BP	MF		DP	SP	DynP	BP	MF
GA	7.6E-10	6.6E-10	7.6E-10	7.6E-10	3.7E-08	HHO	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.5E-01
PSO	7.2E-10	6.7E-10	9.2E-10	6.3E-10	**5.4E-1**	GWA	7.6E-10	7.6E-10	7.6E-10	7.6E-10	2.0E-06
WOA	1.2E-09	9.1E-10	7.6E-10	7.6E-10	3.6E-02	SCA	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.0E-06
Optimization results by different algorithm in literature for welded beam design											
GA[26]	GA [22]	CMVHHO[8]	HWOANM [8]	MVO [8]	GSA[12]	GA-DP	PSO-DP	WOA-DynP	HHO-MDF	GWO-MDF	SCA-MDF
1.728	2.413	1.726	1.880	1.726	1.725	5.926	**1.588**	1.628	1.592	1.598	1.591

Table 10. Speed reducer design statistical results.

Minimum, median, maximum, violations and average processing time results for speed reducer design.											
	DP					DynP					
	Min	Median	Max	Violation /Rerun	Time (sec)	Min	Median	Max	Violation /Rerun	Time (sec)	
GA	2.997E+03	3.021E+03	3.164E+03	0/50	5.338E+01	2.998E+03	3.009E+03	**3.031E+3**	0/50	4.918E+01	
PSO	**2.994E+3**	**3.005E+3**	3.045E+03	0/50	2.964E+00	2.996E+03	3.029E+03	3.047E+03	0/50	2.613E+00	
WOA	3.025E+03	3.140E+03	3.220E+03	0/50	7.303E-01	3.008E+03	3.080E+03	3.188E+03	0/50	5.122E-01	
HHO	**2.994E+3**	3.189E+03	4.415E+03	0/50	1.900E+00	2.998E+03	3.163E+03	4.077E+03	0/50	1.391E+00	
GWO	3.002E+03	3.038E+03	3.064E+03	0/50	8.348E-01	3.012E+03	3.040E+03	3.062E+03	0/50	6.116E-01	
SCA	3.023E+03	3.084E+03	3.139E+03	0/50	7.848E-01	3.038E+03	3.082E+03	3.166E+03	0/50	5.705E-01	
	SP					BP					
GA	3.000E+03	3.007E+03	**3.031E+3**	0/50	5.098E+01	3.000E+03	3.008E+03	3.038E+03	0/50	5.226E+01	
PSO	2.996E+03	3.028E+03	3.180E+03	0/50	3.031E+00	2.996E+03	3.039E+03	3.205E+03	0/50	3.123E+00	
WOA	3.004E+03	3.105E+03	3.341E+03	0/50	4.289E-01	3.011E+03	3.133E+03	3.613E+03	0/50	5.239E-01	
HHO	3.003E+03	3.260E+03	4.320E+03	0/50	1.164E+00	3.010E+03	3.167E+03	3.902E+03	0/50	1.326E+00	
GWO	3.004E+03	3.037E+03	3.066E+03	0/50	5.080E-01	3.007E+03	3.039E+03	3.073E+03	0/50	7.005E-01	
SCA	3.042E+03	3.081E+03	3.144E+03	0/50	4.459E-01	3.013E+03	3.082E+03	3.160E+03	0/50	5.453E-01	
	DF					MDF					
GA	3.000E+03	3.030E+03	3.176E+03	**1/50**	5.480E+01	2.995E+03	3.020E+03	3.159E+03	0/50	5.025E+01	
PSO	3.036E+03	3.045E+03	3.189E+03	0/50	5.097E+00	2.996E+03	3.038E+03	3.188E+03	0/50	5.435E+00	
WOA	3.006E+03	3.764E+03	5.252E+03	0/50	1.515E+00	3.002E+03	3.757E+03	5.129E+03	0/50	1.539E+00	
HHO	3.020E+03	3.966E+03	5.462E+03	0/50	2.015E+00	3.001E+03	3.730E+03	5.379E+03	0/50	2.010E+00	
GWO	3.028E+03	3.086E+03	3.168E+03	0/50	1.660E+00	3.029E+03	3.085E+03	3.108E+03	0/50	1.751E+00	
SCA	3.020E+03	3.044E+03	3.065E+03	0/50	1.827E+00	3.011E+03	3.036E+03	3.060E+03	0/50	1.816E+00	
Significance value results with reference to Wilxocon Test for speed reducer design											
	DP	SP	DynP	BP	MF		DP	SP	DynP	BP	MF
GA	7.6E-10	7.6E-10	7.6E-10	7.6E-10	1.4E-01	HHO	2.4E-04	2.2E-04	9.4E-05	1.3E-02	3.2E-02
PSO	1.7E-04	4.1E-08	2.0E-07	7.6E-10	4.5E-08	GWA	8.5E-04	3.1E-08	7.6E-10	1.2E-09	9.0E-02
WOA	6.7E-10	6.7E-10	7.7E-10	6.3E-10	2.6E-09	SCA	7.6E-10	7.6E-10	7.6E-10	7.6E-10	2.7E-04
Optimization results by different algorithm in literature for speed reducer design											
MBA[27]	WOA [25]	GWO [25]	SSA [25]	PSO [25]	MFO [25]	GA-MDF	PSO-DP	WOA-MDF	HHO-DP	GWO-DP	SCA-MDF
2.9994 E+03	2.9966 E+03	2.9957 E+03	2.9960 E+03	2.9945 E+06	2.9945 E+06	2.9950 E+03	**2.9936 E+3**	3.0021 E+03	**2.9936 E+3**	3.0017 E+03	3.0110 E+03

Table 11. Active filter design statistical results.

Minimum, median, maximum, violations and average processing time results for active filter design											
	DP					SP					
	Min	Median	Max	Violation /Rerun	Time (sec)	Min	Median	Max	Violation /Rerun	Time (sec)	
GA	8.968E-04	5.107E-03	1.000E+10	**3/50**	51.28	2.458E-04	2.877E-03	1.000E+10	8/50	31.15	
PSO	5.979E-04	4.067E-03	1.533E-02	0/50	7.76	1.300E-04	5.144E-02	1.012E+00	0/50	5.07	
WOA	3.391E-04	3.969E-02	1.000E+00	0/50	5.69	5.979E-04	6.330E-03	2.142E-02	0/50	2.76	
HHO	1.034E-04	2.226E-03	6.579E-03	0/50	14.19	2.973E-04	1.918E-03	7.260E-03	0/50	8.13	
GWO	2.973E-04	1.797E-01	9.955E-01	0/50	5.63	2.707E-16	1.170E-01	9.997E-01	0/50	3.09	
SCA	1.030E-03	7.422E-02	1.000E+00	0/50	4.40	7.121E-04	1.081E-01	1.000E+00	0/50	3.13	
DynP						BP					
GA	4.460E-04	1.741E-01	2.822E+00	0/50	51.71	4.226E+07	4.181E-02	1.000E+10	0/50	54.55	
PSO	2.973E-04	3.482E-02	1.411E+00	0/50	5.27	2.134E-04	1.394E-02	4.142E-01	0/50	5.49	
WOA	5.979E-04	5.524E-03	2.284E-02	0/50	2.56	1.213E-03	1.080E-01	1.000E+00	0/50	2.79	
HHO	1.137E-16	1.948E-03	**5.049E-3**	0/50	7.25	1.187E-03	2.030E-03	5.553E-03	0/50	7.52	
GWO	2.973E-04	1.281E-01	4.142E-01	0/50	2.95	9.228E-05	1.334E-01	4.598E-01	0/50	3.47	
SCA	2.974E-04	1.092E-01	1.000E+00	0/50	2.65	8.674E-04	1.087E-01	1.000E+00	0/50	2.93	
DF						MDF					
GA	1.575E-04	1.405E-02	9.556E-01	0/50	31.58	1.338E-04	2.821E-03	1.382E-02	0/50	55.94	
PSO	1.890E-04	9.849E-02	9.153E-01	0/50	23.31	**0**	7.486E-02	9.003E-01	0/50	25.50	
WOA	3.473E-03	1.410E-01	1.000E+00	0/50	11.40	2.472E-03	1.033E-01	1.000E+00	0/50	11.37	
HHO	4.012E-04	2.811E-03	7.770E-03	0/50	17.59	3.616E-05	**1.763E-3**	6.985E-03	0/50	15.10	
GWO	1.363E-03	1.525E-01	9.994E-01	0/50	13.76	1.098E-03	8.812E-02	5.405E-01	0/50	13.91	
SCA	5.880E-04	2.647E-03	9.754E-03	0/50	13.07	2.973E-04	3.418E-03	8.130E-03	0/50	12.80	
Significance value results with reference to Wilxocon Test for active filter design											
	DP	SP	DynP	BP	MF		DP	SP	DynP	BP	MF
GA	**9.5E-1**	4.5E-08	**6.2E-1**	2.9E-04	4.47E-07	HHO	3.8E-09	3.8E-09	4.3E-07	3.6E-08	1.06E-04
PSO	6.3E-10	6.8E-10	7.0E-10	**5.2E-1**	1.40E-09	GWA	3.4E-05	1.4E-01	**6.8E-1**	3.5E-03	2.47E-07
WOA	1.3E-05	1.4E-01	1.4E-05	3.1E-01	1.34E-08	SCA	7.6E-10	1.2E-09	7.6E-10	7.6E-10	2.34E-08
Optimization results by different algorithm in literature for active filter design											
	CS [10]	GA [10]	Conventional Method		GA-MDF	PSO-MDF	WOA-DP	HHO-DynP	GWO-SP	SCA-MDF	
	8.6E-03	7.18 E-02	1.726E+00		1.338E-04	**0**	3.391E-04	1.137E-16	2.707E-16	2.973E-04	

5. Conclusion

In this study, the DF, one of the constraint handling methods, was developed and the MDF approach was improved. The third rule of DF was modified. This modification is based on generating a new individual using both possible solutions that violate the constraints. If this solution is better than the current violations, then it is accepted as a new solution. Therefore, the performance of the improved method was compared with DP, SP, DynP, BP and DF constraint handling methods that exist in the literature. These methods were adapted to GA, PSO, WOA, GWO, HHO and SCA algorithms. Subsequently, benchmark problems involving engineering problems were used to evaluate the performance of algorithms and constraint handling methods with respect to minimum, median, maximum, processing time constraint violation numbers. Moreover, Wilcoxon tests have been employed to measure the strength of results. Consequently, when one examines the results, experimental results demonstrate that, most of the time, MDF is more robust and efficient than DF and some of the other constraint handling techniques. In future studies, this method will be much more developed with different functions and q learning algorithms to manipulate the behavior of the optimization dynamic.

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