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Multiscroll chaotic system with sigmoid nonlinearity and its fractional order form with synchronization application

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ABSTRACT

In this paper, a multiscroll snap oscillator with hyperbolic tangent function is proposed. There is no limitation in the number of scrolls and it can be increased by proper choice of a specific function. The Lyapunov exponents of the proposed system are obtained to testify the chaotic behavior of the system. Fractional order multiscroll system is derived from its integer order model by using the Adams–Bashforth–Moulton algorithm. A new scheme is applied in order to investigate the synchronization of the multiscroll systems. The main objective of the paper is to propose a multiscroll attractor and show that the number of scrolls can be controlled by the only nonlinear function. Such systems are less investigated in the literatures and has many real time applications like image and voice encryption, random number generators, chaos based communication systems and so on.

1. Introduction

Discovering chaotic systems with novel features have attracted much attention in recent years. Most of the literature in this topic deals with chaotic systems with specific equilibria. For example chaotic systems with no equilibrium points [1–4], with stable equilibria [5–8], with curves of equilibria [9–14], with surfaces of equilibria [15–18], and with non-hyperbolic equilibria [19] can be mentioned. Some new chaotic systems with features related to symmetry [20] , and amplitude control [21,22] are also discussed in the literatures. One of the interesting areas of research is chaotic systems generating multi-wing [23] and multi-scroll [24–29] attractors. Also multi-stable systems are very important in nonlinear dynamics. Recently systems with mega-stability [30–34] and extreme multi-stability [35–38] are also discussed as novel chaotic systems, mostly with memristive elements [39–41].

Systems that their equations are derived as third derivative of displacement ($\dddot{x}$) are called jerk systems. They often considered as simplest form of chaotic system. Sprott has announced several cases of jerk systems with two quadratic nonlinearities [42] and three quadratic nonlinearities [43]. A snap or a hyperjerk oscillator is a fourth order differential equation of the form $\frac{d\dddot{x}}{dt} = f(\dddot{x}, \ddot{x}, \dot{x}, x)$ where $\dot{x}$, $\ddot{x}$, $\dddot{x}$, and $\dddot{x}$ represent state variables.

Fractional calculus has history dating back to 17th century. However it found its applications in science and engineering research only in the recent years [44,45]. Many physical systems such as dielectric polarization, electromagnetic waves, and quantum evolution of complex systems exhibits fractional order dynamics and thus fractional order control algorithms are achieving the attention of researchers. Fractional order chaotic systems and their Field Programmable Gate array (FPGA) implementations are discussed in [46–48]. Fractional order Chaotic and Hyperchaotic systems which show both self-excited and hidden attractors for different parameter values are also discussed [48].

There are many multiscroll chaotic attractors in physical systems like the Lorenz system, Rossler system, Elnino model, etc. There have been literatures showing that the generalized form of the Lorenz system can produce multiscroll attractors as in [49] and [50]. The proposed system is a simple hyperjerk systems with a single nonlinearity. Also such multiscroll systems are used in UAV and robotic navigations as seen in the literature [51] and [52]. The importance of the parameters can be seen from the bifurcation diagrams as these parameters controls the chaotic behavior of the system.

Motivated by the above mentioned literatures, in this paper a hyperjerk multi scroll chaotic attractor is introduced. Most of the multi scroll attractors in the literatures are generated using a switching function or a sine function. In this paper a sigmoid (tanh) function is used. A sigmoid function refers to the special case of a logistic function which is real-valued, monotonic, and differentiable having a non-negative

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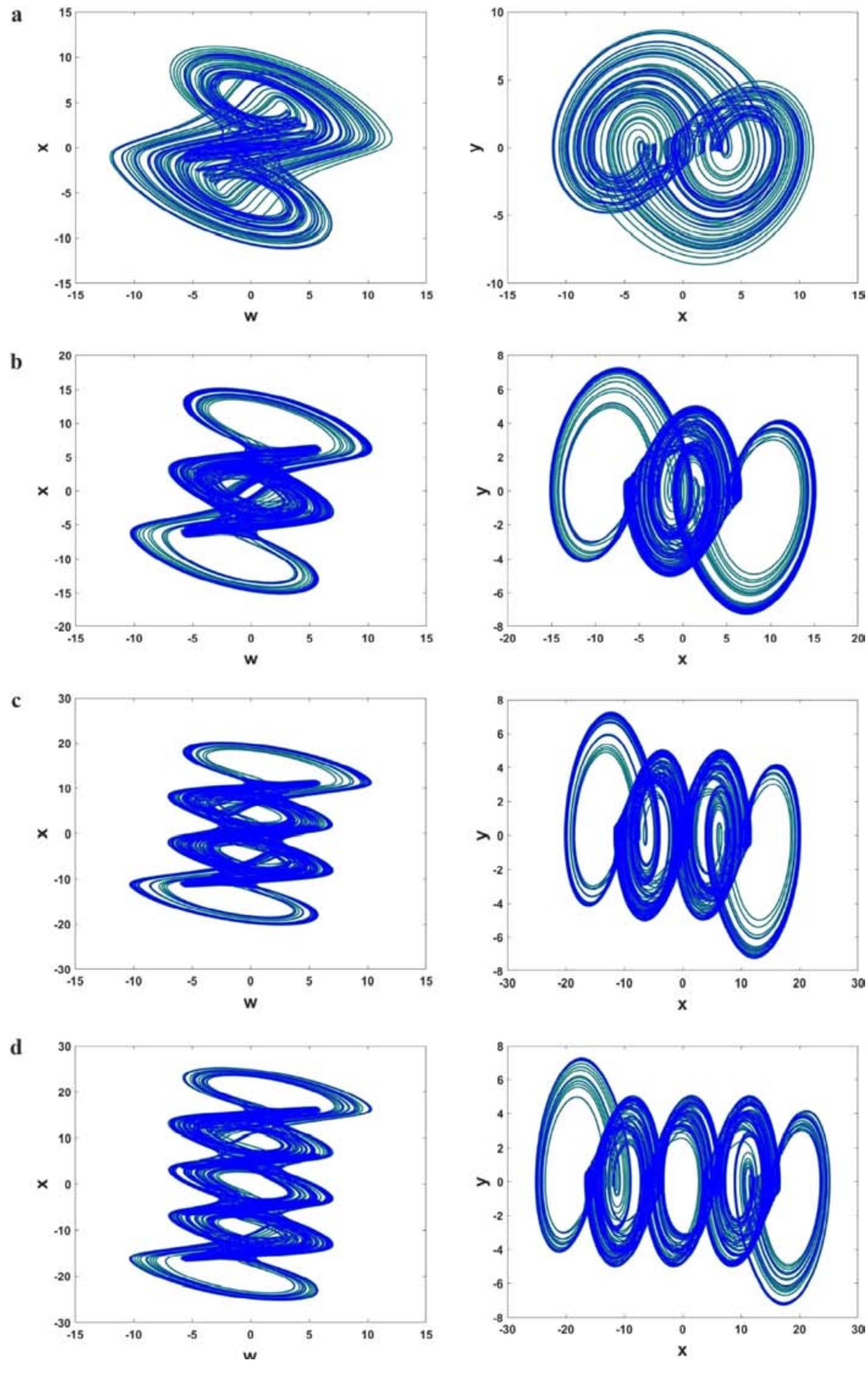

Fig. 1. 2D state portraits (xy and wx planes) of HMSC system: (a) two-scroll, (b) three-scroll, (c) four-scroll, and (d) five-scroll.

first derivative. Dynamic properties of the proposed multi-scroll attractor is presented and investigated. Fractional order multi scroll attractor is then derived using Caputo [44], derivatives and Adam–Bashforth–Moulton (ABM) is used to conduct the numerical analysis. FPGA implementations are presented using Adomian Decomposition Method (ADM).

2. Hyperjerk Multi Scroll Chaotic (HMSC) system

As seen in the literature, multiscroll systems are seen in many physical systems and there are discussions on the existence of multiscroll attractors in generalized Lorenz system [49]. Multiscroll systems are used in navigation control of robotic systems like in [51].

The integer order state space model of the proposed HMSC system can be defined by,

$$\begin{cases} \dot{x} = a_1 y \\ \dot{y} = a_2 z \\ \dot{z} = a_3 w \\ \dot{w} = a_4 x + a_5 y + a_6 z + a_7 w + a_8 M(f) \end{cases} \tag{1}$$

where the multiscroll function $M(f)$ is defined in Table 1.

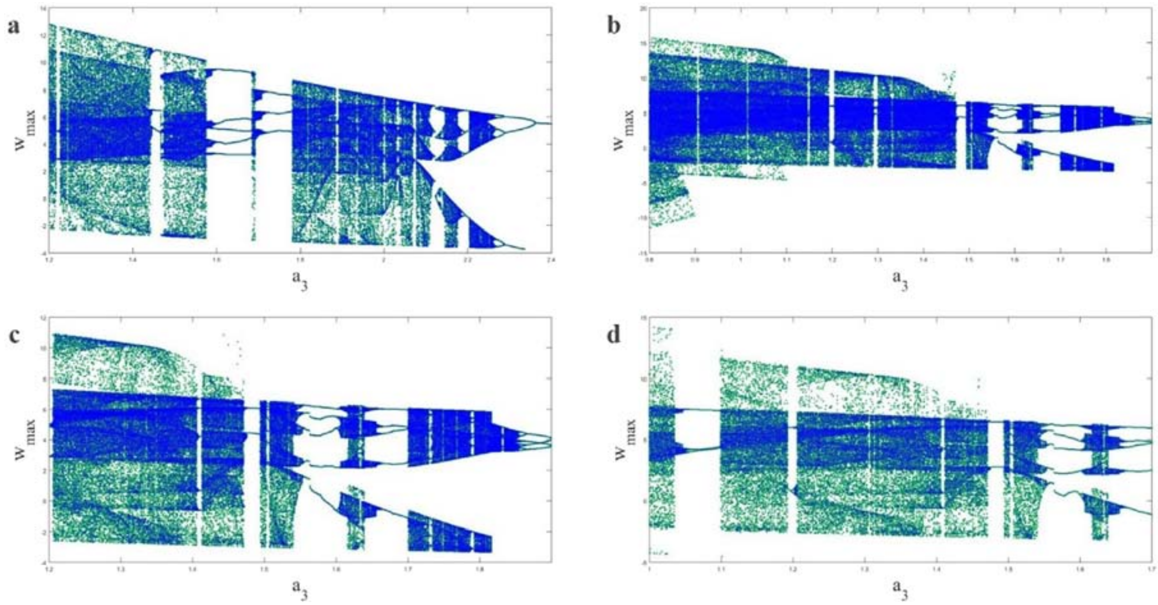

Fig. 2. Bifurcation of HMSC system with a_3: (a) two-scroll, (b) three-scroll, (c) four-scroll, and (d) five-scroll.

Table 1
Multi-scroll functions M(f).

Number of scrolls	$M(f)$
Two	$5\tanh(x)$
Three	$5(\tanh(x-5)+\tanh(x+5))$
Four	$5(\tanh(x-10)+\tanh(x+10)+\tanh(x))$
Five	$5(\tanh(x-15)+\tanh(x+15)+\tanh(x-5)+\tanh(x+5))$

Table 2
Lyapunov exponents of the HMSC system.

Scrolls	Lyapunov exponents
Two	$0.216, 0, -1.215, -3.902$
Three	$0.212, 0, -1.259, -3.853$
Four	$0.108, 0, -1.268, -3.740$
Five	$0.244, 0, -1.513, -3.631$

For the parameters $a_1 = 1.7$, $a_2 = 1.6$, $a_3 = 1.3$, $a_4 = -6$, $a_5 = -5.4, a_6 = -5.2$, $a_7 = -4.9$, $a_8 = 6$ and the initial conditions $(0,-1,0,-5)$, the various chaotic attractors are given in Fig. 1(a–d) .

In order to better study of HMSC system, its Lyapunov Exponents (LEs) are obtained. We have used the well known Wolf's method [53] to calculate the finite time LEs when the run-time is set to 20 000 s Table 2 shows the LEs for the HMSC system.

To investigate the impact of parameters on the HMSC system, we derive the bifurcation plots. By fixing all other parameters to their respective values, we vary the parameter a_3 between $[1.2-2.4]$ for the two-scroll, $[0.9-1.85]$ for three-scroll, $[1.2-1.89]$ for four-scroll and $[1-1.7]$ for the five-scroll HMSC system as can be seen in Fig. 2(a–d). The bifurcation diagram is obtained by plotting local maxima of the state w. The final state values at the end of each iteration of the parameter serves as the initial state for the next iteration.

3. Fractional order HMSC (FOHMSC)

In this section we derive the fractional order HMSC system using the Caputo fractional derivative [44] defined by,

$$D^q x = J^{m-q} x^m \tag{2}$$

where m is the integer closest to q with $m > q$ and J^α is the αth order Riemann–Liouville integral operator given by

$$J^\alpha y = \frac{1}{\Gamma(\alpha)} \int_0^1 (t-\tau)^{\alpha-1} y(\tau)\, d\tau \tag{3}$$

where Γ is the gamma operator.

Using the qth order Caputo fractional derivative (2) and (3), the FOHMSC system is derived as,

$$\begin{cases} D^{q_x} x = a_1 y \\ D^{q_y} y = a_2 z \\ D^{q_z} z = a_3 w \\ D^{q_w} w = a_4 x + a_5 y + a_6 z + a_7 w + a_8 M(f) \end{cases} \tag{4}$$

where $a_1 = 1.7$, $a_2 = 1.6$, $a_3 = 1.3$, $a_4 = -6$, $a_5 = -5.4$, $a_6 = -5.2$, $a_7 = -4.9$, $a_8 = 6$ and the multiscroll function $M(f)$ is taken from Table 1. For the numerical analysis of the FOHMSC system, there are three main approaches namely frequency-domain method, Adomian Decomposition Method (ADM) and Adams–Bashforth–Moulton (ABM) algorithm. The frequency-domain method is not always reliable in detecting chaos behavior in nonlinear systems [54]. In this section we use the predictor–corrector method of Adams–Bashforth–Moulton (ABM) whose convergence and accuracy is more.

Let us define a fractional order dynamical system with order q as,

$$D^q x = f(t, x), \quad 0 \le t \le T \tag{5}$$

where $x^k(0) = x_0^k$ for $k \in [0, n-1]$ and T is the finite time.

Eq. (5) is similar to the Volterra integral equation [55] given by,

$$x(t) = \sum_{k=0}^{n-1} x_0^k \frac{t^k}{k!} + \frac{1}{\Gamma(q)} \int_0^1 \frac{f(\tau_1, x)}{(t-\tau)^{1-q}} d\tau_1 \tag{6}$$

where $h = \frac{T}{N}$, $t_n = nh$ where $h \in [0, N]$. and $\tau_1 = \frac{\tau}{h}$.

The discrete form of (6) can be defined as,

$$x(n+1) = \sum_{k=0}^{n-1} x_0^k \frac{t^k}{k!} + \frac{h^q}{\tau(q+z)} f(t_{n+1}, x_n'(n+1)$$
$$+ \frac{h^q}{\Gamma(q)} \sum a_{j,n+1} f(t_j, x_n(j)))$$

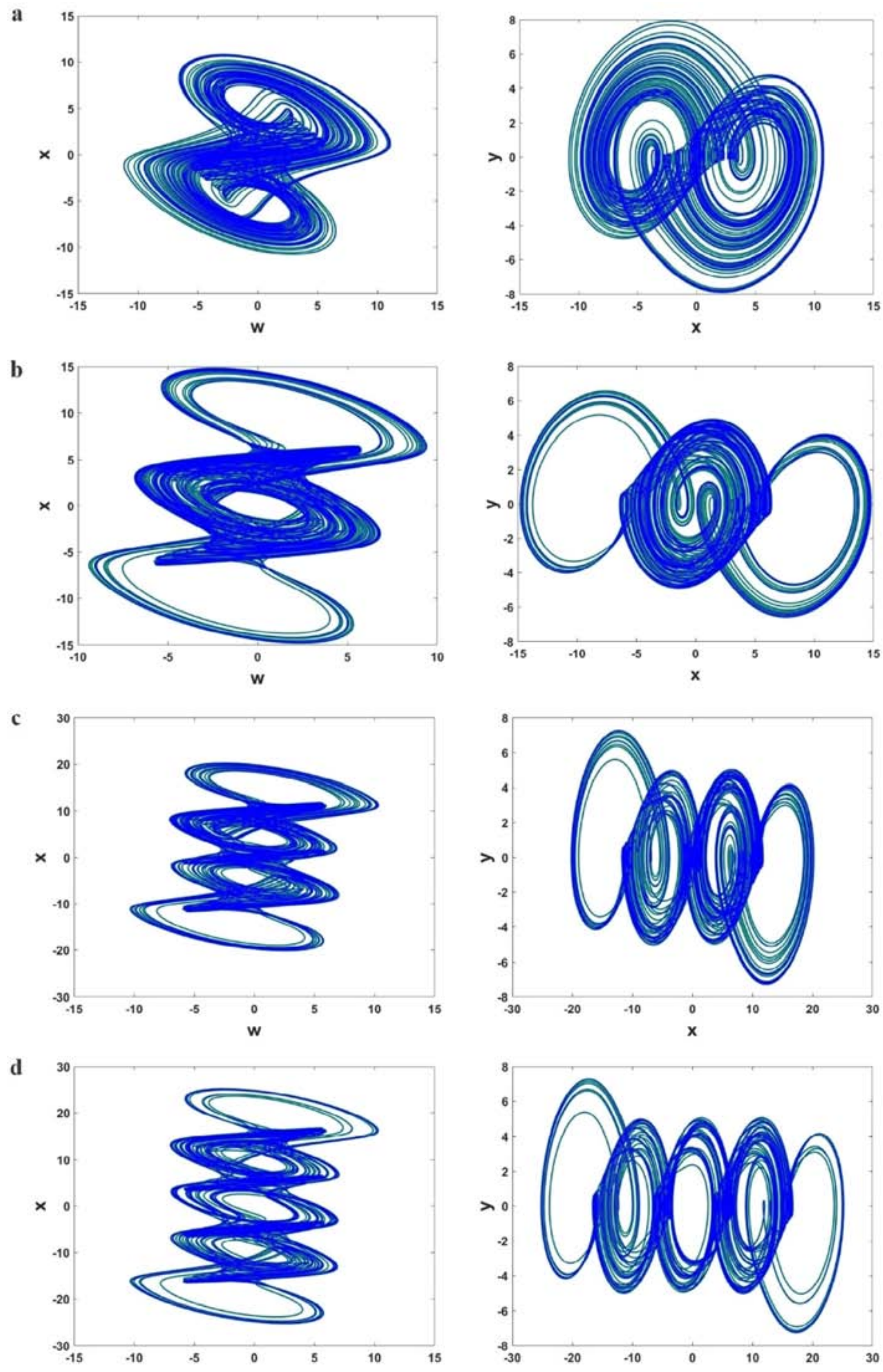

Fig. 3. 2D state portraits (xy and wx planes) of FOHMSC system: (a) two-scroll, (b) three-scroll, (c) four-scroll, and (d) five-scroll.

$$a_{j,n+1}=\begin{cases} n^{q+1}-((n-q)\,n+1)^{q+1} & j=0\\ (n-j+2)^{q+1}+(n-j)^{q+1}-2(n-j+1)^{q+1} & 1\le j\le n\\ 1 & j=n+1\end{cases}$$

$$x_n'(n+1)=\sum_{k=0}^{n-1}x_0^k\frac{t^k}{k!}+\frac{1}{\Gamma(q)}\sum b_{j,n+1}f\left(t_j,x_n(j)\right)$$

$$b_{j,n+1}=\frac{h^q}{q}\left((n-j+1)^q-(n-j)^q\right) \tag{7}$$

Using the above definitions (6) and (7), the FOHMSC system can be defined as,

$$x(n+1)=x(0)+\frac{h^{q_x}}{\Gamma\left(q_x+2\right)}\left(\left(a_1y'(n+1)\right)+\sum_{j=0}^{n}\chi_{1,j,n+1}\left(a_1y(j)\right)\right)$$

$$y(n+1)=y(0)+\frac{h^{q_y}}{\Gamma\left(q_y+2\right)}\left(\left(a_2z'(n+1)\right)+\sum_{j=0}^{n}\chi_{2,j,n+1}\left(a_2z(j)\right)\right)$$

$$z(n+1)=z(0)+\frac{h^{q_z}}{\Gamma\left(q_z+2\right)}\left(\left(a_3w'(n+1)\right)+\sum_{j=0}^{n}\chi_{3,j,n+1}\left(a_3w(j)\right)\right)$$

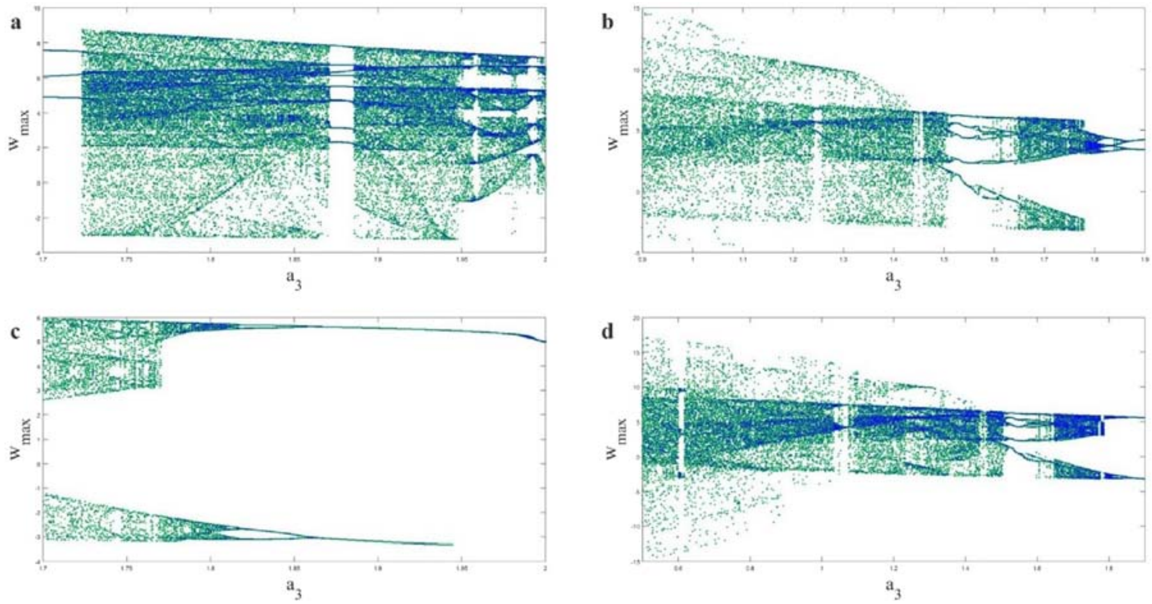

Fig. 4. Bifurcation of FOHMSC (state w) system with a_3: (a) two-scroll, (b) three-scroll, (c) four-scroll, and (d) five-scroll.

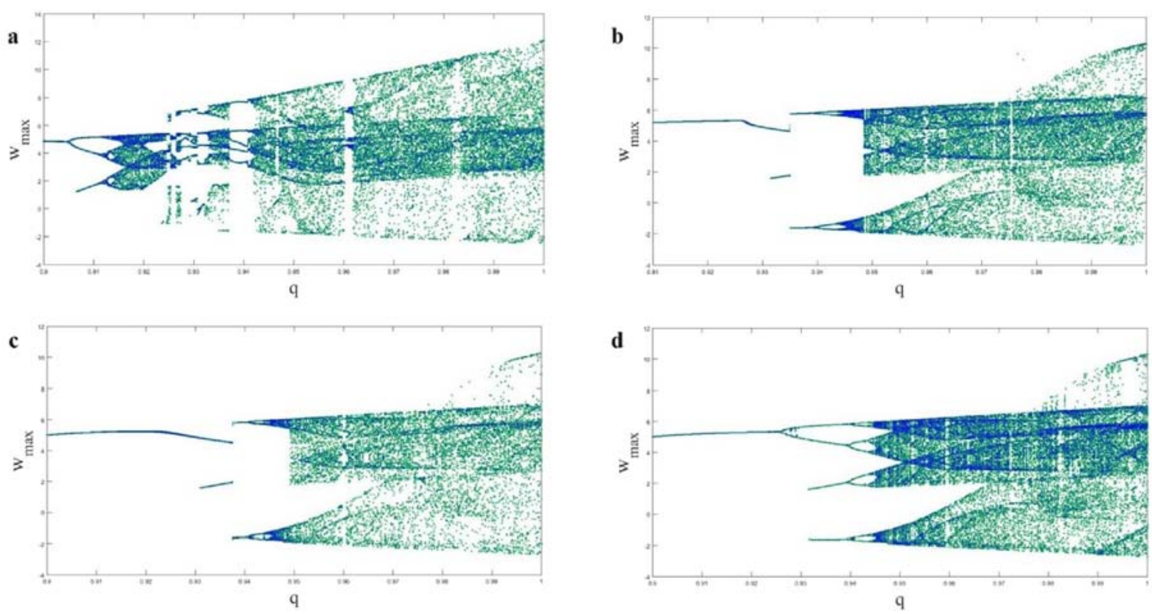

Fig. 5. Bifurcation of FOHMSC system with q: (a) two-scroll, (b) three-scroll, (c) four-scroll, and (d) five-scroll.

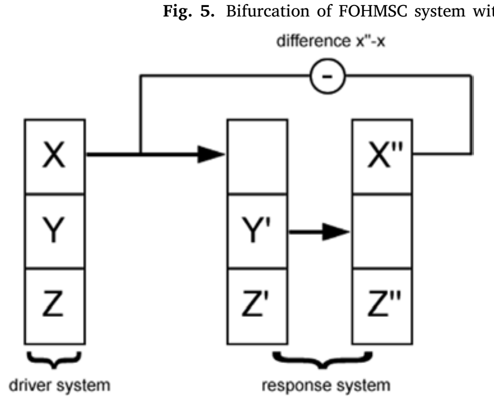

Fig. 6. The block diagram of P–C Synchronization [56].

$$w(n+1) = w(0) + \frac{h^{q_w}}{\Gamma\left(q_w+2\right)} \left(\begin{array}{l} \left(a_4 x'(n+1) + a_5 y'(n+1) + a_6 z'(n+1) + a_7 w'(n+1) + a_8 M(f)\right) + \\ \sum_{j=0}^{n} \chi_{4,j,n+1} \left(a_4 x(j) + a_5 y(j) + a_6 z(j) + a_7 w(j) + a_8 M(f)\right) \end{array} \right) \tag{8}$$

where

$$x'(n+1) = x(0) + \frac{1}{\Gamma\left(q_x+2\right)} \left(\sum_{j=0}^{n} \theta_{1,j,n+1} \left(a_1 y(j)\right) \right)$$

$$y'(n+1) = y(0) + \frac{1}{\Gamma\left(q_y+2\right)} \left(\sum_{j=0}^{n} \theta_{2,j,n+1} \left(a_2 z(j)\right) \right)$$

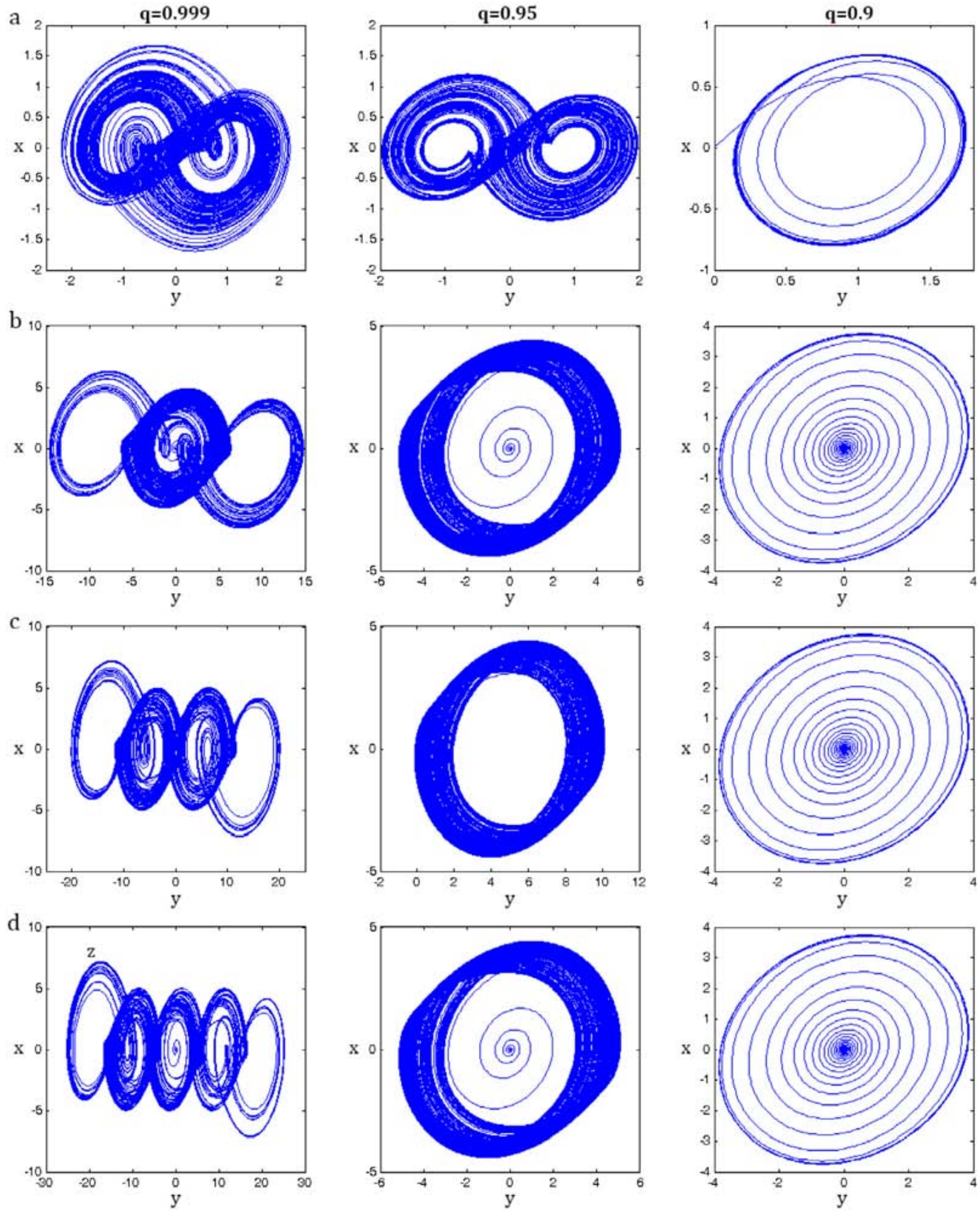

Fig. 7. 2D state portraits (xy planes) of FOHMSC system with different q values (q = 0.999, q = 0.95 and = 0.9): (a) two-scroll, (b) three-scroll, (c) four-scroll, and (d) five-scroll.

$$z(n+1) = z(0) + \frac{1}{\Gamma(q_z+2)}\left(\sum_{j=0}^{n}\theta_{3,j,n+1}\left(a_3 w(j)\right)\right)$$

$$w'(n+1) = w(0) + \frac{1}{\Gamma(q_w+2)}\left(\sum_{j=0}^{n}\theta_{4,j,n+1}\left(a_4 x(j) + a_5 y(j) + a_6 z(j) + a_7 w(j) + a_8 M(f)\right)\right)$$

$$\chi_{i,j,n+1} = \begin{cases} n^{q+1} - (n-q)(n+1)^{q+1} & j = 0 \\ (n-j+2)^{q+1} + (n-j)^{q+1} - 2(n-j+1)^{q+1} & 1 \le j \le n \\ 1 & j = n+1 \end{cases}$$

$$\theta_{i,j,n+1} = \frac{h^q}{q}\left((n-j+1)^q - (n-j)^q\right), \quad 0 \le j \le n \quad for \;\; i = 1,2,3,4. \tag{9}$$

where q takes the respective values of q_x, q_y, q_z, q_w depending on the state variables. The 2D phase portraits of the FOHMSC system solved with ABM are shown in Fig. 3(a–d) .

The Lyapunov exponents of the FOHMSC system are derived using the Wolfs algorithm [53] by using the fractional order predictor–corrector [57] solver fde12 [51] in place of the ode solvers [58]. The fde12 solver is modified accordingly so that we could use the fractional orders for one state variable whereas the other two states remains in the integer order. The Lyapunov exponents of the FOHMSC system are numerically found as in Table 3 for a fixed value of $q = 0.99$.

To show the importance of the parameters and fractional order q the bifurcation plots are investigated. Firstly a_3 is varied by fixing

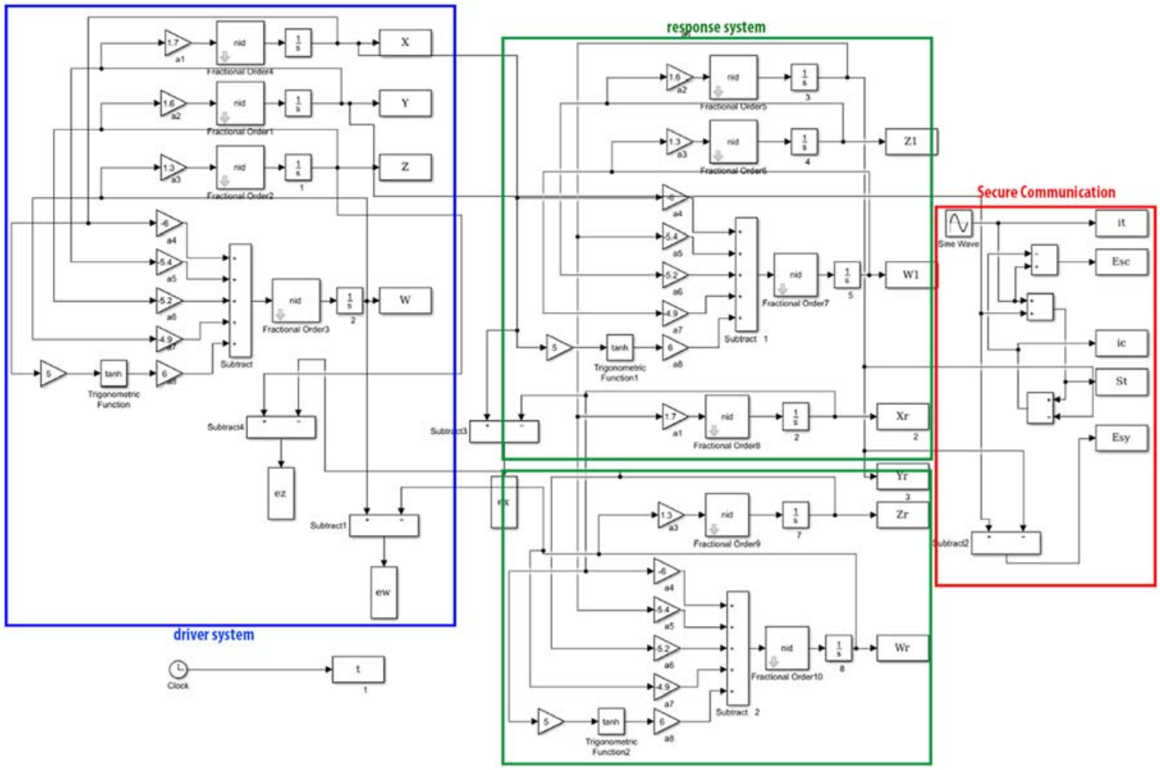

Fig. 8. Matlab Simulink model of two scroll FOHMSC system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

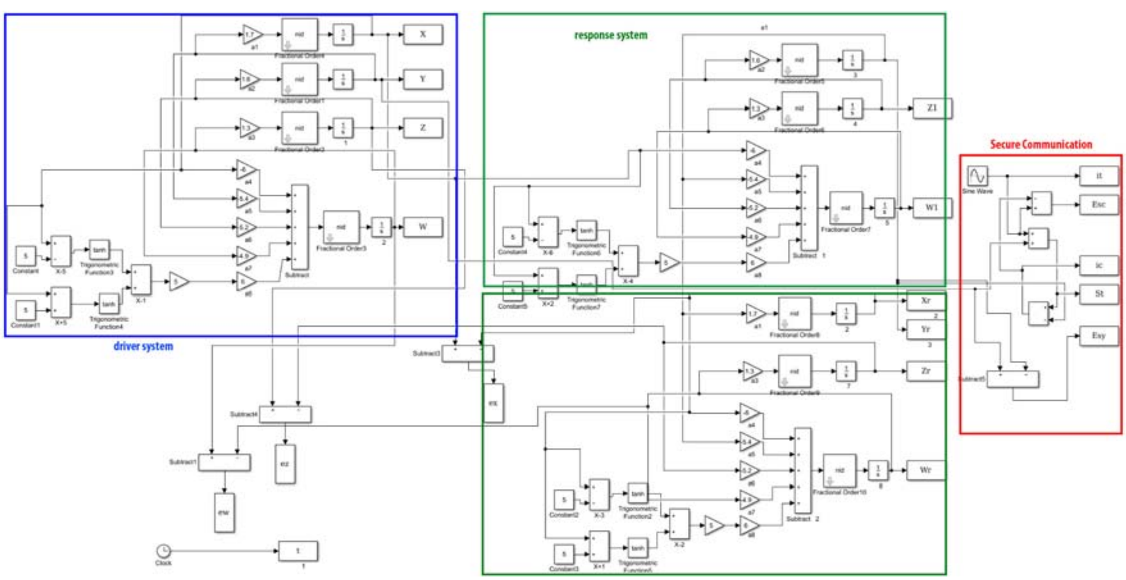

Fig. 9. Matlab Simulink model of three scroll FOHMSC system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3
Lyapunov exponents of the FOHMSC system.

Scrolls	Lyapunov exponents
Two	0.224, 0, −1.237, −3.931
Three	0.248, 0, −1.281, −3.902
Four	0.125, 0, −1.276, −3.812
Five	0.261, 0, −1.531, −3.667

all the other parameters to their respective values of $a_1 = 1.7$, $a_2 = 1.6$, $a_4 = -6$, $a_5 = -5.4$, $a_6 = -5.2$, $a_7 = -4.9$, $a_8 = 6$ and fractional order $q = 0.995$. Fig. 4(a–d) shows the bifurcation of the FOHMSC system with parameter a_3 varied between $[1.7 - 2]$, $[0.9 - 1.9]$, $[1.7-2]$ and $[0.55 - 1.85]$ for two-scroll, three-scroll, four-scroll and five-scroll FOHMSC systems respectively.

Similar to the parameter bifurcation, the bifurcation plots for the commensurate fractional order q of the FOHMSC system are obtained. Fig. 5(a–d) shows the bifurcation of FOHMSC system with fractional order q. The initial conditions for the first iteration are taken as

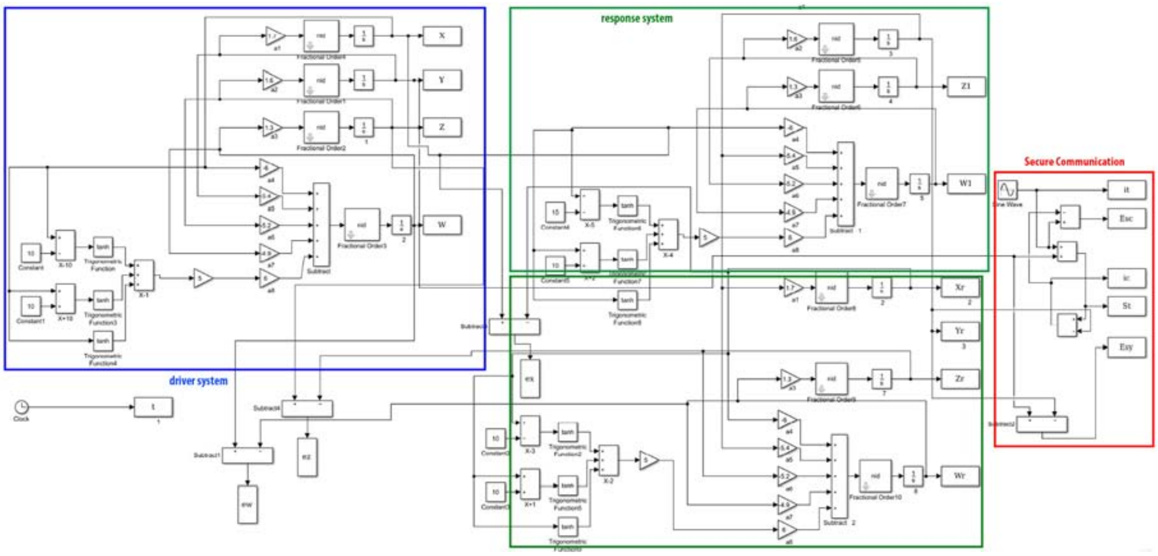

Fig. 10. Matlab Simulink model of four scroll FOHMSC system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

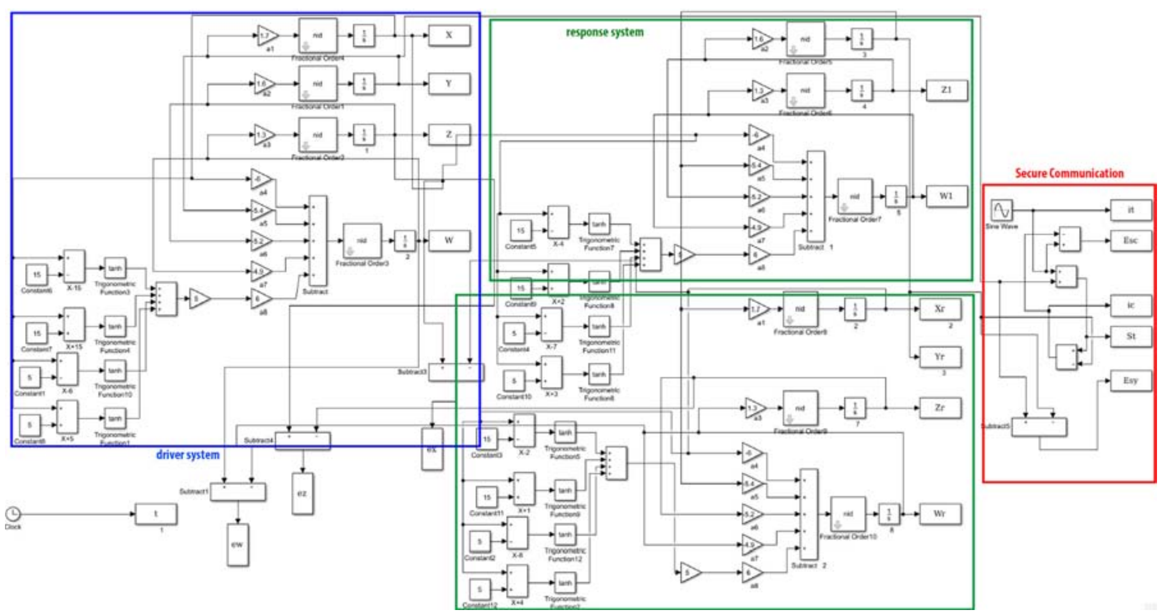

Fig. 11. Matlab Simulink model of five scroll FOHMSC system. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(1, 1, 1, 1) and the fractional order is varied between [0.9 – 1]. Similar to the HMSC system, FOHMSC system takes period doubling route to chaos with some periodic windows in chaotic area.

4. Synchronization of the fractional order chaotic systems

In Fig. 6, Pecora–Carroll (P–C) [56] synchronization block diagram method is implemented for the proposed fractional order chaotic systems. As it can be seen in Fig. 6, X, Y and Z state variables stimulate to the subsystem directly. The response system is divided into two subsystems. X state variable of the first subsystem is taken from the driver system. Y state variable of the second subsystem is taken from the first subsystem. With the P–C method, two identical chaotic systems with different initial conditions are synchronized. The driver system's X state variable and the response system's X″ state variable's difference is zero, which indicate that the two chaotic systems are synchronized.

In Fig. 7, phase portraits of FOHMSC with the different values of fractional order (q) are given. Different behavior of the system can be seen in phase portraits with respect to changing the values of q. In the condition $0.9 < q < 0.999$, all of the chaotic systems exit from chaos. These situations are not demonstrated in Fig. 7. When $q = 0.9$, all of the systems approach to the chaos. When $q = 0.95$, two scroll chaotic systems are in the chaos situation. Other three chaotic systems are quite close to the chaos. When $q = 0.999$, all of the systems are chaotic.

The P–C method is applied to the FOHMSC in the Matlab Simulink environment and the Simulink models are given in Figs. 8–11. In Matlab simulink implementation, fractional order chaotic system is modeled with Nid function. As it is seen in the Figs. 8–11, secure communication is also provided in all of the synchronization implementation. As shown in Figs. 8–11, the blue frame driver system, the green frames response system and the red frame show the secure communication application. In secure communication, Y (driver system) and Yr (response system) state variables are used. According to secure

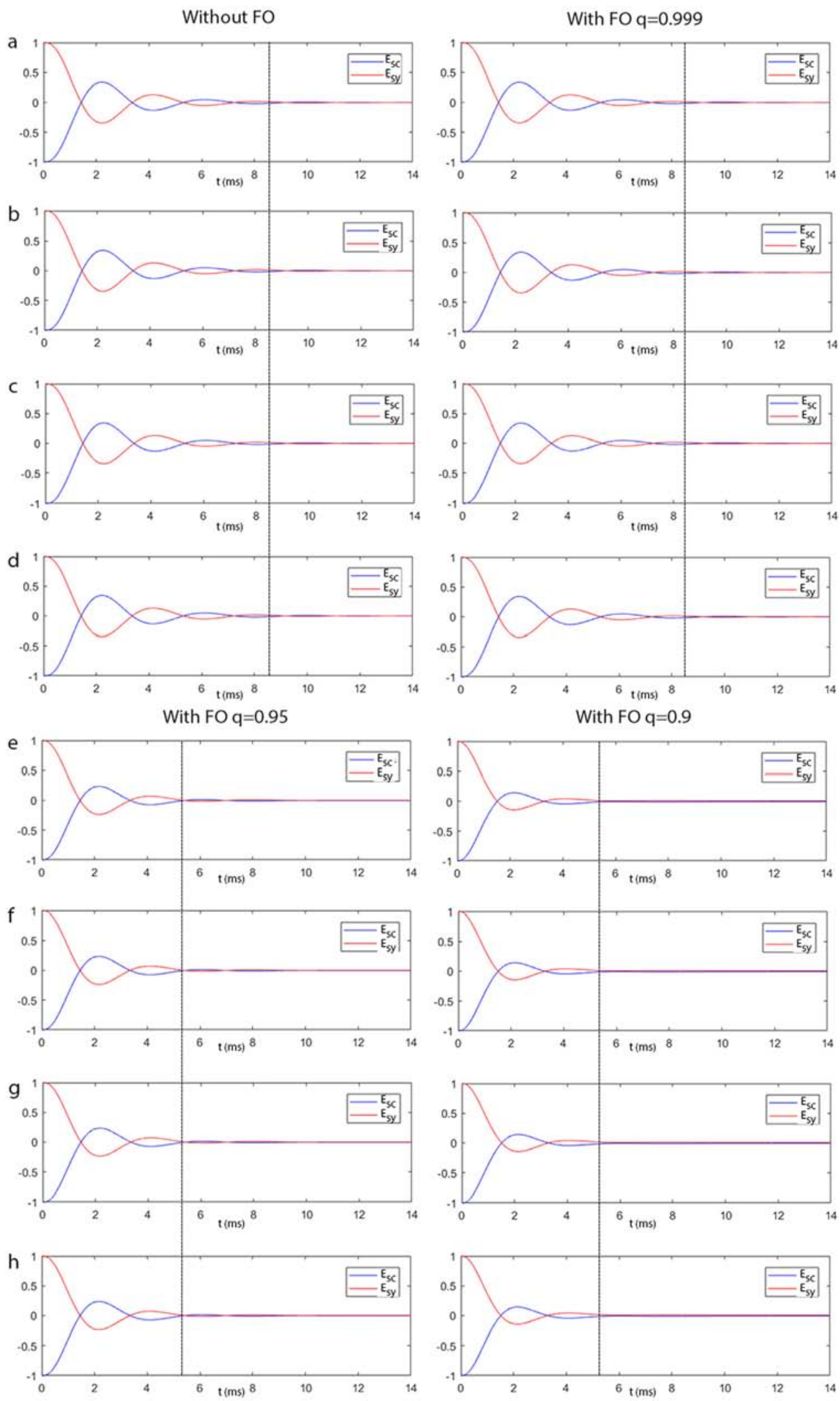

Fig. 12. Synchronization error of HMSC and FOHMSC (q = 0.999, q = 0.95 and q = 0.9) systems. (Esc = error secure communication, Esy = error sync y state variable): (a) and (e) two-scroll, (b) and (f) three-scroll, (c) and (g) four-scroll, (d) and (h) five-scroll.

communication, sinus signal (it) that has 10 V-amplitude value is sent through the FOHMSC. Sinusoidal signal (it) with the chaotic signal obtained from the Y state variable of the driver system are combined and obtained encrypted signal (St). The chaotic signal obtained from the Yr state variable of the response system subtract from Encrypted signal (St). Thus received signal (ic) is obtained. The difference between the sent (it) signal and the received (ic) signal is indicate the error of

Table 4
Synchronization times of HMSC and FOHMSC system.

FO (q)	Scrolls			
	Two	Three	Four	Five
	Time (ms)			
Without FO	8.4	8.4	8.4	8.4
0.999	8.3	8.3	8.3	8.3
0.95	5.3	5.3	5.3	5.3
0.9	5.5	5.5	5.5	5.5

secure communication (Esc). The difference of Yr and Y state variables gives the synchronization error (Esy).

In Fig. 12, graphics of synchronization error by time in HMSC and FOHMSC (q = 0.999, q = 0.95 and q = 0.9) which is synchronized with P–C method are given. In Fig. 12a–d. in without FO, graphics of error signals which occurred as a result of secure communication and synchronization of HMSC is given. In Fig. 12a–d. in with FO q = 0.999, Fig. 12e–h, with FO q = 0.95 and q = 0.9, graphics of error signals which occurred as a result of secure communication and synchronization of FOHMSC is given. In all of the graphics, the error which occurred as a result of synchronization error (Esy) and the error which occurred as a result of secure communication (Esc) created a mirror image owing to the harmony between each other. In Fig. 12a–d, in without FO, HMSC synchronized in 8.4 ms. In Fig. 12a–d, in with FO q = 0.999, FOHMSC are synchronized in 8.3 ms. In which q = 0.95, synchronized in 5.3 ms. In which q = 0.9, synchronized in 5.5 ms. Scroll numbers in chaotic systems did not affect the sync duration. In this case, the fourth status, in which q = 0.95 value is used, is synchronized 3.1 ms earlier than the first status, in which fractional order is not used.

In Table 4, sync duration of the systems that summarizes the graphs in Fig. 12 is given. According to the chart, the fastest sync duration for all of the chaotic systems is when q value is 0.95 with 5.3 ms.

5. Conclusion

In this paper, a new multiscroll snap oscillator was proposed which can be chaotic. The bifurcation diagrams of the system are plotted and the Lyapunov exponents are obtained. In the next step, it was shown that the number of scrolls can be controlled by proper choice of the nonlinear function. Then, Adams–Bashforth–Moulton algorithm was used to derive the fractional order model of the multiscroll system. Finally, Pecora–Carroll method of synchronization was used to synchronize the fractional order multiscroll systems and determine the proper time of synchronization. It was shown that while the number of scrolls does not affect the time of synchronization, decreasing the fractional order decrease the synchronization time.

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