An exponential jerk system: Circuit realization, fractional order and time delayed form with dynamical analysis and its engineering application ☆

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Abstract

Simple dynamical systems are of interest always. In this paper we propose a simple jerk system with one exponential nonlinearity. Dynamical properties of the proposed system are investigated. To show the practical realisability of the proposed system we implement the exponential jerk system using off the shelf components. Fractional order and time delays are considered as complex analysis patterns of nonlinear systems. We investigate the fractional order time delayed exponential jerk system.For numerical analysis we use the modified Adomian Decomposition Method. To show the engineering importance of the proposed system, we derive a pseudo random number generator based on it.Various test results are presented to show the randomness of the system.

Keywords:
Exponential jerk system (EJS), fractional-order and time-delayed chaotic systems, dynamic analyses, electronic circuit implementation, random number generator (RNG).

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1. Introduction

After the discovery of chaos as a field of study, various chaotic systems have been reported in the literature. But the construction of dynamical systems with simple mathematical expressions are still a challenge even though the most popular Lorenz system [1] is often used as a best example for chaotic systems, it cannot be said that Lorenz system is simple in implementing or realizing it in analog or digital circuits. Much simpler chaotic systems like the one Rossler announced in 1976 with only one quadratic nonlinearity [2] and the one announced by him a decade later with much simpler dynamics [3] can be said to be mathematically simpler models. Many more much simplex chaotic systems are announced with only six terms and one quadratic nonlinearity [4]. Recently there is an interest in especial chaotic systems. Systems with hidden attractors [5, 6, 7], no equilibrium [8, 9], stable equilibria [10], curves of equilibria [11], surfaces of equilibria [12, 13, 14], non-hyperbolic equilibria [15], amplitude control [16, 17, 18, 19], multistability [20, 21, 22], extreme multistability [23, 24, 25, 26], megastability [27, 28, 29, 30].

The simplest of such systems are the Jerk systems. It is the lowest derivative for which an ODE with smooth continuous functions can give chaos [31]. The jerk system is the system that cover the 3rd derivative of x and processes the acceleration change rate [32]. Sprott has proposed a three quadratic nonlinear system in his work [33]. Also, there are also studies on A simple chaotic hyperjerk or snap system in the literature [34].The circuit implementation of Jerk systems has been presented in some works [35]. In the recent years, fractional order systems and their applications have been studied [36]. Studies on the fractional nonlinear system with different control design are continued[37]. Fractional order memristor without equilibrium point and hyper chaotic systems are presented in the literature[38]. There are also works in the literature on fractional order chaotic systems without new equilibrium point [39].

Time delayed differential equations play an important role in most of the engineering applications [40]. Their stability analysis of differential equations delays have also been discussed in [41]. Synchronization of such time delayed systems is a major complex problem and many synchronization schemes have been discussed in literature [42]. A time delayed chaotic system was obtained from logistic chaotic map [43]. A parameter identification problem for a general time delayed chaotic system is considered and analyzed in [44]. A novel time delayed chaotic system with hidden attractors was discussed in [45]. Another parameter identification problem for determining the unknown parameters of time delay chaotic system was investigated in [46].Various methods of fractional order time delayed synchronization with sliding mode [47], active control [48], ring connection synchronization [49], lag synchronization [50] and generalized synchronization [51] was also discussed in the literatures.

In this article, the exponential jerk system (EJS) and the fractional order time delayed exponential jerk system (FOTDEJS) system and dynamical analysis are introduced in Section 2. A time delayed version of the fractional order jerk system is derived assuming different time delays in the third state equation. Bifurcation of the time delayed fractional order system with time delay and parameters are investigated. The EJS system circuit implementation has been done in Section 3. A new RNG algorithm has been designed using the developed chaotic system in Section 4. In the last section, conclusions are given.

2. Fractional order time delayed exponential jerk system (FOTDEJS) and its dynamic properties

2.1. Exponential jerk system (EJS)

In this work an exponential jerk system is obtained from the jerk system introduced in [52]. The proposed EJS differs from the class of chaotic flows discussed in [53] as those systems are dependent on discontinuities which is difficult when implementing in hardware circuits. The introduced EJS system is given by the dimension model defined as,

$$\begin{aligned} \dot{x} &= -y \\ \dot{y} &= -z \\ \dot{z} &= -x - bz + ae^{y} \end{aligned} \tag{1}$$

where $a = 0.05$ and $b = 0.7$. The initial conditions are $[0, 0.1, 1]$. The 2D phase portraits of the EJS system on x-z and y-z planes as given Figure 1.

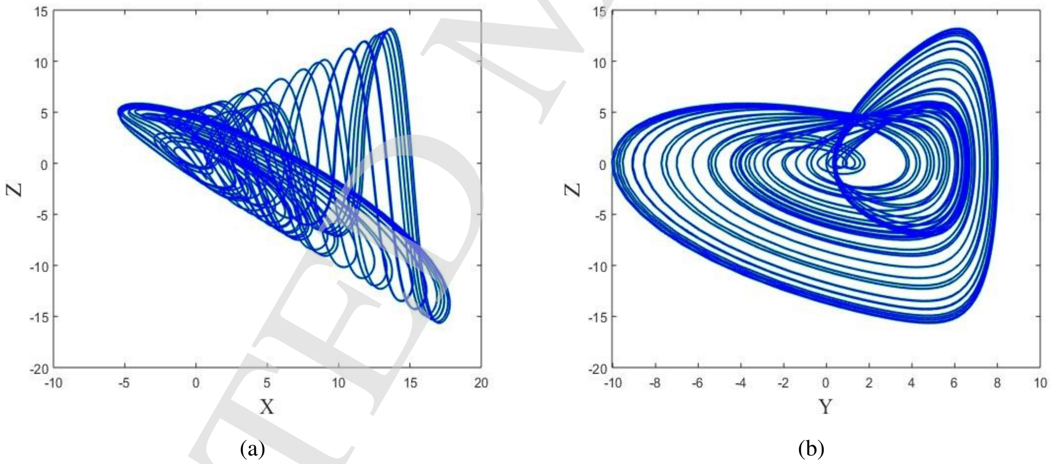


Figure 1: 2D phase portraits of the EJS system: (a) $x - z$; (b) $y - z$.

2.2. Fractional order time delayed exponential jerk system (FOTDEJS)

Performing simulations with time-delay or fractional order is a complex task and combining both is a challenging area of research. The fractional differential equations tend to lower the dimensionality of the differential equations and introducing delay in differential equations makes it infinite-dimensional and even a single ordinary differential equation with delay can exhibit chaos [54]. Because of their wide applicability, study of fractional DDEs is of theoretical and practical interest. To the best of our knowledge dynamical analysis of time delayed fractional order systems are less investigated. In this section we propose the fractional order time delayed exponential jerk system derived from FOEJS by introducing different time delays in the state equation. Introducing different time delays with time delay in a fractional order jerk system is nowhere discussed in the literature. The FOTDEJS state equations are given by,

$$\begin{aligned} D^{q_x}x &= -y \\ D^{q_y}y &= -z \\ D^{q_z}z &= -x_{\tau_1} - bz_{\tau_2} + ae^{y_{\tau_3}} \end{aligned} \tag{2}$$

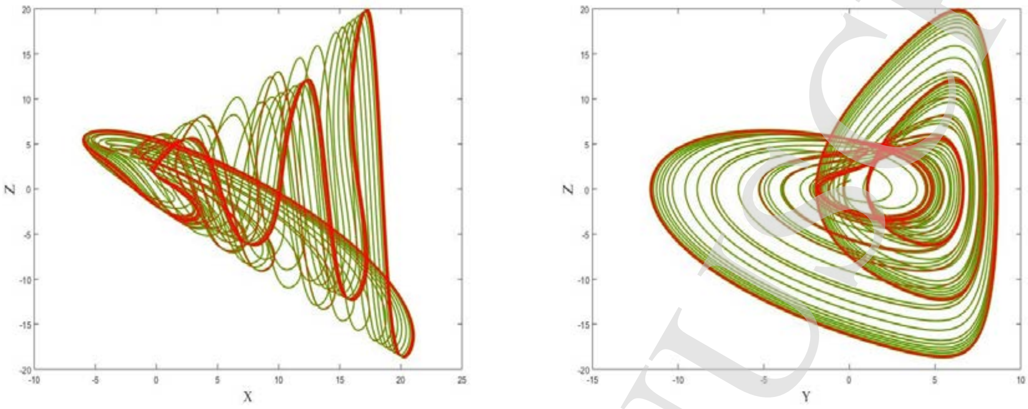
where $x_{\tau_1} = x(t-\tau_1), y_{\tau_3} = y(t-\tau_3), z_{\tau_2} = z(t-\tau_2)$ and $a = 0.05, b = 0.7$. The three time delays are kept at $\tau_1 = 0.1, \tau_2 = 0.1$ and Adomian polynomials for the linear terms can be calculated as $\tau_3 = 0.01$. To numerically solve the FOTDEJS, we adopt the ADM method for time delayed fractional order systems [55]. The $A_1^{j+1} = I_t^q(-A_2^j); A_2^{j+1} = I_t^q(-A_3^j)$ and the first six Adomian polynomials for the non-linear term (e^y) can be derived as,

$$\begin{aligned}
A_3^1 &= I_t^q(-A_1^0 - bA_3^0 + ae^{A_2^0}) \\
A_3^2 &= I_t^q(-A_1^1 - bA_3^1 + aA_2^1 e^{A_2^0}) \\
A_3^3 &= I_t^q(-A_1^2 - bA_3^2 + a\left[e^{A_2^0}\left(A_2^2 + \tfrac{1}{2}\left(A_2^1\right)^2\right)\right]\tfrac{\Gamma(2q+1)}{\Gamma^2(q+1)}) \\
A_3^4 &= I_t^q(-A_1^3 - bA_3^3 + a\left[e^{A_2^0}\left(A_2^3 + \tfrac{1}{6}\left(A_2^1\right)^3 A_2^1 A_2^2\right)\right]\tfrac{\Gamma(3q+1)}{\Gamma(q+1)\Gamma(2q+1)}) \\
A_3^5 &= I_t^q(-A_1^4 - bA_3^4 + a\left[e^{A_2^0}\left(\begin{array}{l} A_2^1 A_2^3 \\ +\tfrac{1}{24}\left(A_2^1\right)^4 + \tfrac{1}{2}\left(A_2^2\right)^2 \\ +\tfrac{1}{2}\left(A_2^1\right)^2 A_2^2 + A_2^4 \end{array}\right)\right]\tfrac{\Gamma(4q+1)}{\Gamma(q+1)\Gamma(3q+1)}) \\
A_3^6 &= I_t^q(-A_1^5 - bA_3^5 + a\left[e^{A_2^0}\left(\begin{array}{l} A_2^5 + \tfrac{1}{2}A_2^1\left(A_2^2\right)^2 \\ +A_2^1 A_2^4 + \tfrac{1}{120}\left(A_2^1\right)^5 \\ +\tfrac{1}{2}\left(A_2^1\right)^2 A_2^3 \end{array}\right)\right]\tfrac{\Gamma(5q+1)}{\Gamma(q+1)\Gamma(4q+1)})
\end{aligned} \tag{3}$$

More general form of the discrete Adomian decomposed FOTDEJS can be given by

$$\begin{aligned}
\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{bmatrix} &= I_t^q \begin{bmatrix} A_1^0 & A_1^1 & A_1^2 & A_1^3 & A_1^4 & A_1^5 & A_1^6 \\ A_2^0 & A_2^1 & A_2^2 & A_2^3 & A_2^4 & A_2^5 & A_2^6 \\ A_3^0 & A_3^1 & A_3^2 & A_3^3 & A_3^4 & A_3^5 & A_3^6 \end{bmatrix} \\
&\times \begin{bmatrix} 1 & \frac{h^q}{\Gamma(q+1)} & \frac{h^{2q}}{\Gamma(2q+1)} & \frac{h^{3q}}{\Gamma(3q+1)} & \frac{h^{4q}}{\Gamma(4q+1)} & \frac{h^{5q}}{\Gamma(5q+1)} & \frac{h^{6q}}{\Gamma(6q+1)} \end{bmatrix}^T
\end{aligned} \tag{4}$$

Using the equation (3), the FOTDEJS is numerically solved and Figure 2 shows the 2D state portraits of the FOTDEJS.



(a) (b)

Figure 2: 2D phase portraits of the FOTDEJS: (a) $x - z$; (b) $y - z$.

2.3. Dynamic properties of the FOTDEJS

The dynamic properties of the FOTDEJS such as equilibrium points, Eigen values, Lyapunov exponents and bifurcation with time delay and parameters are derived and discussed in this section.

2.3.1. Equilibrium points

Since at the equilibrium points the time delay τ doesnt affect the system as the system does not change with time at the equilibrium points and hence the FOTDEJS shows only one equilibrium point at origin (O) as like the EJS system at $[a, 0, 0]$.

$$[(J_0 + e^{-\lambda\tau} J_\tau) - \lambda I] = 0 \tag{5}$$

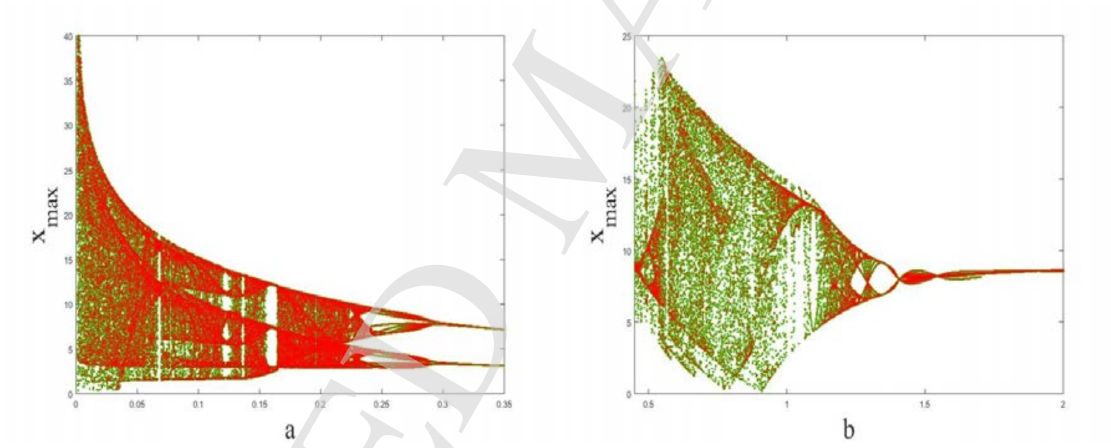
where J_0 is the normal Jacobian matrix and J_τ is the Jacobian matrix for the time delay τ. As the equilibrium points of delayed and non-delayed systems are the same, the Jacobian matrix J_0 and J_τ also remains same. The characteristic equation of the FOTDEJS is $\lambda^3 + 0.7\lambda^2(e^{0.1\lambda} + 1) + \lambda(0.05e^{0.2\lambda} + 0.1e^{0.1\lambda} + 0.05)$ $+3e^{0.2\lambda} + 3e^{0.1\lambda} + e^{.3\lambda} + 1$. The characteristic equation has an absolute minimum of '1' at $\lambda = 0$, so the characteristic equation has no real solutions. To find complex solutions, we write the Eigen value in complex form $\lambda = a + ib$ and we need to find out whether these equations can have solutions with positive values of the real part a so that the system exhibits chaotic oscillations. Note that the characteristic values come in complex-conjugate pairs. We can therefore restrict our attention to positive values of imaginary part b. Solving the equation, we get $a = 0.0127$ and $b = 0.6635$ hence the Eigen values are complex conjugate pair with positive real part (saddle focus) and thus the FOTDEJS exhibits chaotic oscillations.

2.3.2. Lyapunov exponents

There are various algorithm proposed based on chaos synchronization for the estimation of Lyapunov exponent of time delayed dynamical systems [56]. In this paper we adopted the technique employing the synchronization of identical systems coupled by linear negative feedback mechanism [57] for finding the exact Lyapunov exponents of the FOTDEJS. The calculated Lyapunov exponents are $L_1 = 0.1229, L_2 = 0, L_3 = -0.8222$.

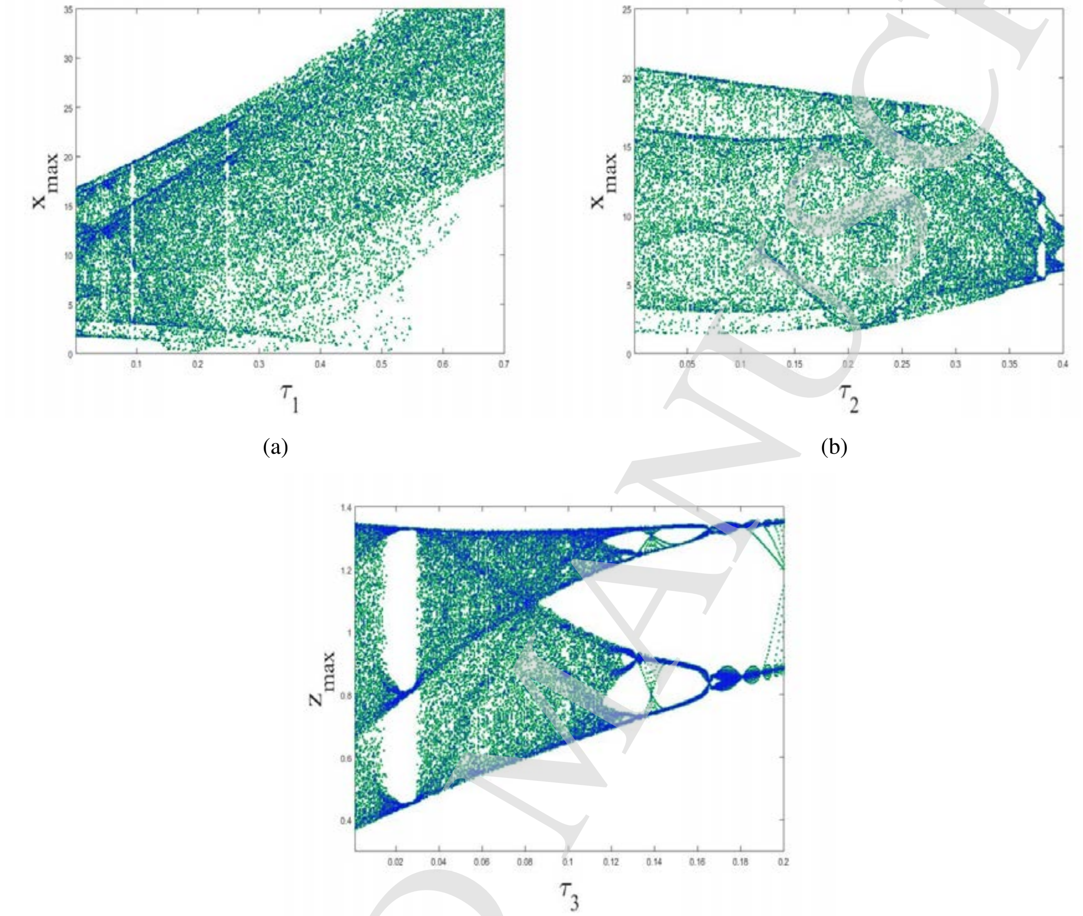
2.3.3. Bifurcation

To understand the parameter dependence of the FOTDEJS, we derive and investigate the bifurcation plots. We discuss about bifurcation of the FOTDEJS with parameters (a, b), time delays (τ_1, τ_2, τ_3) and commensurate fractional order (q). Figure 3 shows the bifurcation plots of FOTDEJS with parameters a and b. Figure 4 shows the impact of time delays on the bifurcation of the FOTDEJS. More specifically time delays τ_2 and τ_3 show a routine inverse period doubling way to exit chaotic regime and shows chaotic regimes for $0 < \tau_2 < 0.37$ and $0 < \tau_3 < 0.14$. Figure 5 shows the bifurcation of the FOTDEJS system with fractional order q.



(a) (b)

Figure 3: Bifurcation plots of FOTDEJS system with parameters a and b.



(a) (b) (c)

Figure 4: Bifurcation of FOTDEJS system with time delays τ_1, τ_2, τ_3.

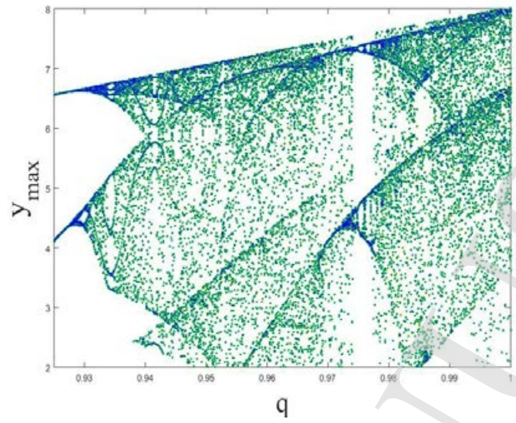


Figure 5: Bifurcation of FOTDEJS system with fractional order q

3. Circuit implementation of exponential jerk system (EJS)

There are many papers in the literature related to electronic ciecuit designs on the oscilloscope [58, 59, 60, 61, 62, 63, 64, 65, 66, 67]. In this section, the a circuit design of exponential jerk system (1) are differently implemented on the oscilloscope as real-time engineering application. There are very little works in the literatur related to chaotic systems have exponential term. The done works are implemented in simulation programmes because of very diffucult on the oscilloscope. In this paper, we did the electronic circuit implementation on the oscilloscope.

The 3D EJS is described like in below:

$$\begin{cases} \dot{x} = -y \\ \dot{y} = -z \\ \dot{z} = -x - bz + ae^{y} \end{cases} \tag{6}$$

where a, b are parameters and x, y, z are state variables. The parameters values are set to $a = 0.05$ and $b = 0.7$. The initial state of EJS is chosen as $(0, 0.1, 1)$.

The EJS's some signal values in this work has been exhibited high values more than -15V and +15V. To avoid saturation property of electronic components signal values of EJS must be reduced by a linear scaling of the variables. So, EJS are firstly scaled before electronic circuit application. There are different methods related to linear scaling in literature[68, 69]. We didn't use any electronic components for this process.

Let $X = x/2, Y = y/4$ and $Z = z/2$ for scale process and then setting the original variables x, y, z instead of the variables X, Y, Z the scaled EJS becomes like in below.

$$\left\{\begin{array}{l}\dot{x}=-y\\ \dot{y}=-z\\ \dot{z}=-x-bz+ae^{y}\end{array}\right. \Rightarrow \left.\begin{array}{r}X=\frac{x}{2}\\ Y=\frac{y}{4}\\ Z=\frac{z}{2}\end{array}\right\} \Rightarrow \begin{array}{c}x=2X\\ y=4Y\\ z=2Z\end{array} \tag{7}$$

Finally, scaled EJS are given by

$$\begin{array}{l}\dot{X}=-2Y\\ \dot{Y}=-\frac{Z}{2}\\ \dot{Z}=-X-bZ+\frac{a}{2}e^{4Y}\end{array} \tag{8}$$

The electronic circuit of the scaled EJS is designed in OrCAD-PSpice simulation programme. The circuit schematic of EJS is like in Figure 6. The EJS circuit is designed for parameter $a=0.05$, $b=0.7$ and initial conditions $x(0)=0, y(0)=0.1, z(0)=1$.

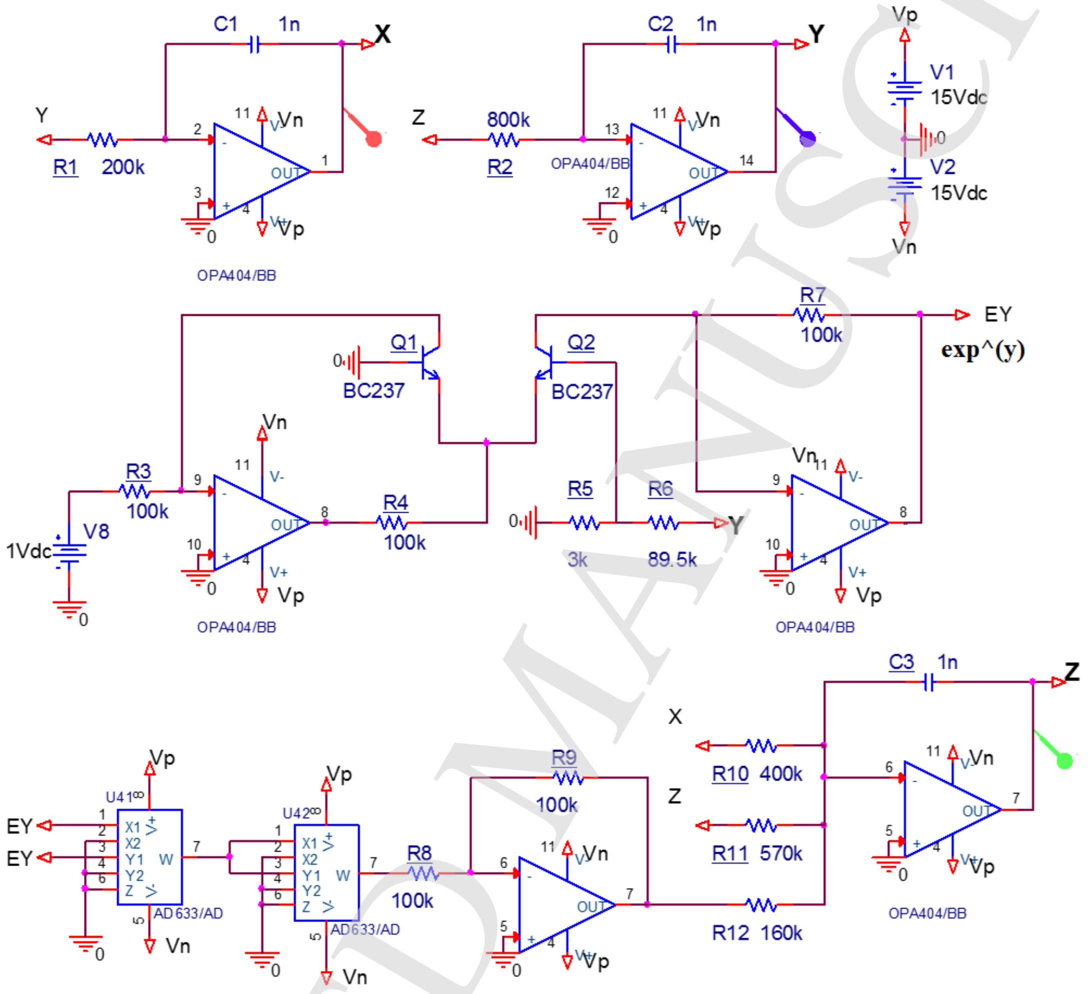


Figure 6: The electronic circuit schematic of the scaled chaotic EJS

R1 = 200 Kohm, R2 = 800 Kohm, R3 = R4 = R7 = R8 = R9 = 100 Kohm, R5 = 3 Kohm, R6 = 89.5 Kohm, R10 = 400 Kohm, R11 = 570 Kohm, R12 = 160 Kohm, C1 = C2 = C3 = 1 nF, Vn = -15 V, Vp = 15 V were used. The experimental circuits of the chaotic EJS are implemented on electronic card in Figure 7 and in Figure 8 with oscilloscope.

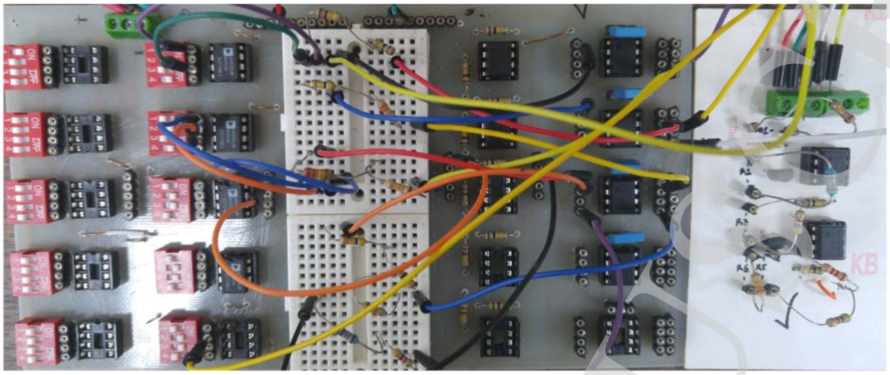


Figure 7: The experimental circuit of the EJS circuit

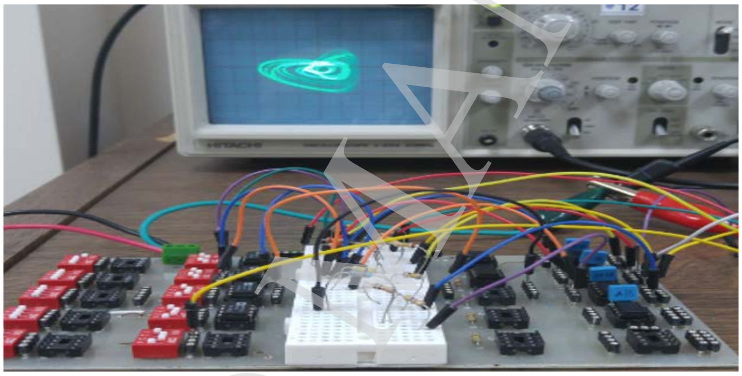
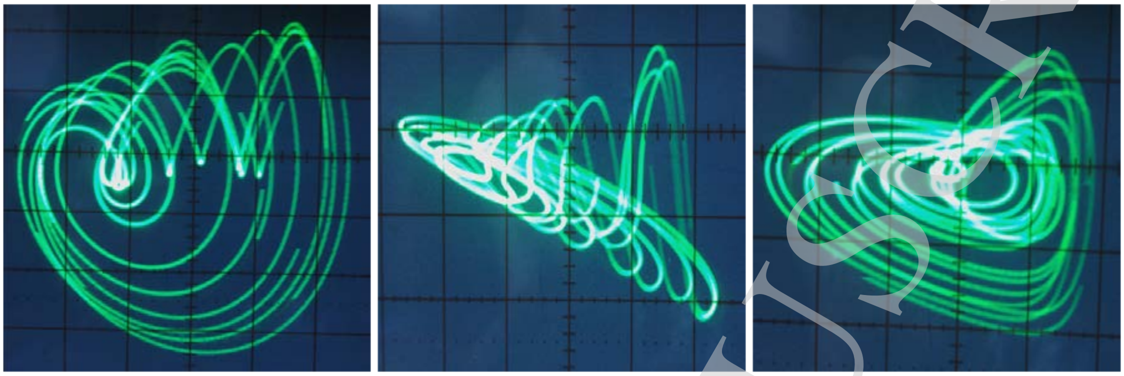


Figure 8: The experimental circuit of the chaotic EJS circuit with oscilloscope

The oscilloscope outputs of chaotic EJS are seen in Figure 9 for x-y, x-z and y-z planes. The real-time application outputs show that results of the chaotic EJS which was obtained on MATLAB verify the oscilloscope results.



(a) (b) (c)

Figure 9: The phase portraits of scaled of the EJS system on the oscilloscope: (a) $x - y$; (b) $x - z$; (c) $y - z$.

4. FOTDEJS based random number generator algorithm and NIST 800-22 Test Results

4.1. The RNG Algorithm Design

RNG (Random Number Generator) is widely used for random number generation in encryption applications. Chaotic systems are often preferred in RNG designs due to their rich random dynamic properties. In this section, a new RNG design is presented with the fractional time delay FOTDEJS chaotic system which was introduced and analyzed. The block diagram for the RNG algorithm is shown in Figure 10. When the block diagram is examined, binomial coefficient calculation and memo function are used in solving the chaotic system used in RNG algorithm. Firstly, the fractional orders parameters value is entered in the algorithm. The fractional order values q1, q2 and q3 are set as 0.99. Then the system parameters and initial conditions of the chaotic system are set. The binomial coefficent values (cp1, cp2, cp3) to be used in numerical analysis of the system are calculated. The initial condition values are determined. In the analysis of the system, the time delay value τ_x, τ_y, τ_z is used as 0.01 in three phases.

After all these values are entered and the binomial coeeficient values are calculated, numerical analysis of the chaotic system is performed. The pseudo code used in the analysis of the system is shown in Algorithm 1. In the numerical analysis of the system, The Grunwald-Letnikov approach is used [70, 71]. The Memo function is used in the analysis of the system together with the binomial coefficient values and the parameters it receives. In the analysis of the system, sampling values are obtained by subtracting the value returned by the memo function from the value obtained from the chaotic system. It is seen that the determined delay values are used to solve the system in the pseudo code in Algorithm 1. The resulting float values from each phase are converted to 32-bit binary form. It is added to the random number sequence by selecting 16 bits from the low-valued part with high sensitivity from the 32-bit number sequence. This process continues until 1 M. random number is generated from x, y and z phases. It is necessary to test whether the generated random bit sequences have sufficient randomness. In the next section, the NIST randomness tests are applied to the generated random bit sequences.

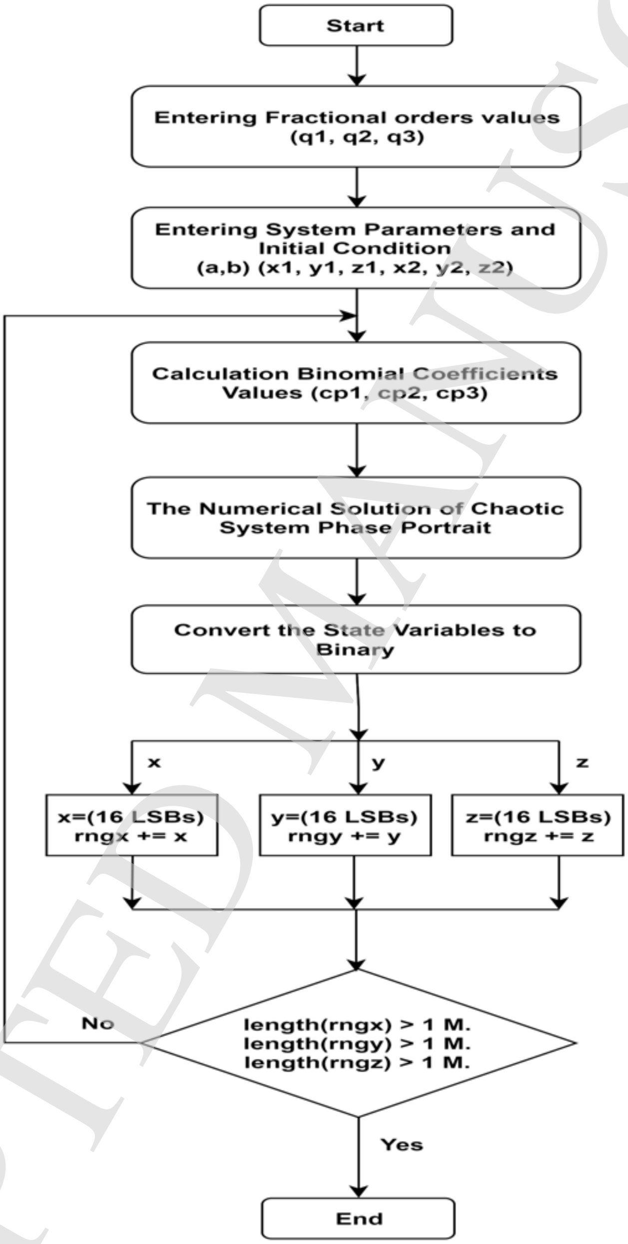


Figure 10: The Block Diagram of RNG Algorithm

Algorithm 1 The Pseudo Code of Numerical Solution of Chaotic System

$memo \rightarrow (memoryfunction)$
for $i = 3 : 1000000$ **do**
    $x(i) = -y(i - \tau_x) * h^{q1} - memo(x, c1, i);$
    $y(i) = -z(i - \tau_y) * h^{q2} - memo(y, c2, i);$
    $z(i) = (-x(i) - b * z(i - \tau_z) + a * exp(y(i))) * h^{q3} - memo(z, c3, i);$
end for

4.2. NIST 800-22 Test Results

NIST 800-22 tests [72] are widely used and accepted tests for testing the randomness of random bit sequences. In the NIST 800-22 tests, the numerical sequences are subjected to many different statistical tests. In order to prove that random number sequences have sufficient randomness, it is necessary to pass all the tests. Table 1 shows the NIST test results of the random number sequences of the three phases obtained from the RNG. When the results are examined, it has been determined that the number sequences obtained from all the phases pass through all the tests. In order for the generated number sequences to have sufficient randomness, the number sequences must pass all the tests. If at least one of the tests remains, the result is that it does not have enough randomness. In this case, by changing the steps and parameters in the RNG algorithm, the production of new number arrays will be performed and the generation of random number arrays with sufficient randomness throughout the tests will be ensured. In this case, the steps and parameters in the RNG algorithm will be modified to produce new random number arrays.It will be possible to produce number arrays with sufficient randomness and passed all of the tests.

Table 1: The NIST-800-22 Test Results

Statistical Tests	P-value (x)	P-value (y)	P-value (z)	Result
Frequency (Monobit) Test	0,64695	0,12066	0,82587	Successful
Block-Frequency Test	0,87566	0,89165	0,44068	Successful
Cumulative-Sums Test	0,91979	0,10046	0,99853	Successful
Runs Test	0,89649	0,40970	0,18953	Successful
Longest-Run Test	0,39561	0,82845	0,66091	Successful
Binary Matrix Rank Test	0,08506	0,15223	0,44661	Successful
Discrete Fourier Transform Test	0,11875	0,89777	0,55699	Successful
Non-Overlapping Templates Test	0,14747	0,26688	0,07416	Successful
Overlapping Templates Test	0,14517	0,57505	0,51318	Successful
Maurer's Universal Statistical Test	0,08262	0,92764	0,19881	Successful
Approximate Entropy Test	0,28833	0,23466	0,60473	Successful
Random-Excursions Test (x = -4)	0,90443	0,88084	0,82572	Successful
Random-Excursions Variant Test (x = -9)	0,12737	0,66818	0,54248	Successful
Serial Test-1	0,94292	0,01365	0,76610	Successful
Serial Test-2	0,72716	0,02472	0,62842	Successful
Linear-Complexity Test	0,28833	0,45037	0,52848	Successful

5. Conclusions

A new exponential jerk system with single nonlinearity is presented and investigated. Circuitry for the prosed system is presented and implemented using readily available off the shelf components. Fractional order time delayed model of the proposed system is derived by including constant time delays. Modified Adomian Decomposition method is used for numerical analysis of the proposed fractional order time delayed system. Finally to show the engineering importance of the proposed system we derive a random number generator using the fractional order exponential jerk system and NIST-800 test results are presented to show the randomness.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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