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RESEARCH PAPER

A New Six-Term 3D Unified Chaotic System

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Abstract

In this study, four different 3D five-term chaotic flows are unified and a novel six-term 3D unified chaotic system with three nonlinearities is introduced. Firstly, the theoretical system via an electronic circuit is realized, and then the basic dynamical properties of the proposed unified chaotic system are numerically and analytically analyzed, i.e., sensitivity to initial conditions, equilibrium points, eigenvalues, Kaplan–Yorke dimensions, dissipativity, Lyapunov exponents and bifurcation diagrams. Investigation results clearly present that this is a new unified chaotic system and earns further detailed disquisition.

Keywords Sprott B chaotic flow · Sprott C chaotic flow · van der Schrier–Mass chaotic system · Munmuangsaen–Srisuchinwong chaotic system · Unified chaotic system

1 Introduction

Chaos theory is used to explain the apparently complex behaviors of simple, linear and well-behaved mathematically defined nonlinear systems. The essential characteristics of chaotic systems are sensitive dependence on initial values and have infinite number of different periodic responses (Miladi et al. 2015). Nonlinear dynamical systems including simple mathematical equations can exhibit chaos that has rich signal trajectories. Because of extreme sensitivity to initial conditions, the chaotic behaviors occur over long-term unpredictability (Stollenwerk et al. 2015). Therefore, chaos is defined as the sufficient conditions of unstable behavior in deterministic dynamical systems.

The researchers have discovered many chaotic systems after the first chaotic attractor was presented by Lorenz (1963). The differential equations of the Lorenz system come from a simplified model of atmospheric convection, and it shows two-scroll graphics. A simple continuous-time three-dimensional chaotic system, which constitutes chemical reaction, was pointed out by Rössler (1976) and a four-dimensional hyperchaotic system by Rössler (1979). A double-scroll attractor was determined from an electronic circuit named Chua's circuit by Matsumoto (1984). Sprott (1994) explored 19 simpler third-order chaotic flows, which have either two nonlinearities with five terms (Sprott A–E) or one nonlinearity with six terms (Sprott F–S). Chen and Ueta (1999) proposed a novel three-dimensional chaotic attractor, which is called as Chen chaotic system. Lü et al. (2002a) developed a continuous-time chaotic attractor that likes the Lorenz and Chen systems as a new chaotic system. Some new five-term chaotic systems are also discovered (Sprott 1997a, b; Van der Schrier and Maas 2000; Munmuangsaen and Srisuchinwong 2009; Chang and Kim 2013; Huang 2013; Yu et al. 2013; Maaita et al. 2015; Pham et al. 2019; Wang et al. 2019). The trend of searching for novel chaotic attractors still continues (Gotthans et al. 2016; Jafari et al. 2016; Pham et al. 2016; Kengne et al. 2017; Pham et al. 2017; Wang et al. 2017; Mobayen et al. 2018; Xiong et al. 2018; Huynh et al. 2019;

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Singh and Roy 2019), and many more will be found because of their potential applications especially in secure communication (Durdu et al. 2015; Nwachioma et al. 2019) and encryption (Ullah et al. 2018; Mobayen et al. 2019).

Lately, the researchers are busy with a new investigation that combining some chaotic attractors. For instance, the generalization form of the Lorenz, Chen and Lü systems was given by Lü et al. (2002b) as a novel chaotic system. It is the first unified chaotic system. Then, a new chaotic system was proposed with combining the unified system with the Rössler system (Gao et al. 2006). Another unified chaotic system was introduced according to Vanecek and Celikovsky criterion, and its linear and adaptive feedback synchronizations were applied by Pan et al. (2010a). A novel three-scroll unified chaotic system, which contains Lorenz-like and Chen-like subsystems as two extremes of its parameter spectrum, was introduced by Pan et al. (2010b). The dynamical behaviors of a novel unified chaotic system that denotes a two-family chaotic system containing the Lorenz and Chua systems with a new constructed joint function were presented (Elhadj and Sprott 2010). Besides, a sixth-order unified hyperchaotic thermal convection loop in both pure fluid layers and fluid-saturated porous media was designed by Sheu et al. (2009). Furthermore, a novel hyperchaotic system was introduced by adding a nonlinear controller as a new state into the three-dimensional unified chaotic system and its control was investigated by using two different control methods (Wang and Zhao 2010). Nowadays, the fractional-order of the first unified chaotic system was analyzed, its circuit diagram was designed, and its control and synchronization were applied with the active control method (Li et al. 2019).

Due to their simplicity, five-term chaotic flows have significant importance. Therefore, their dynamics, control, synchronization and secure communication application were investigated in some papers. Generalized Sprott B (Feng and Wei 2015), Sprott C (Wei and Yang 2012) and Sprott E (Oliveira and Valls 2016) chaotic systems were presented, and their dynamical properties were analyzed. The control of generalized Sprott B chaotic flow was implemented with delayed feedback control method by Feng and Wei (2015), and the control of Sprott E chaotic flow was used with distributed delay feedback control method by Xu and Wu (2015). Synchronization of chaos between Sprott B and Sprott C systems was applied via linear coupling (Liu and Fei 2006). Adaptive control method was applied for the projective synchronization of the five-term 3D chaotic system with a nonlinear quadratic exponential term (Huang 2013), and an adaptive control scheme was designed for the projective synchronization between different chaotic systems (Hamri and Ouahabi 2017). Electronic circuits were constructed for the secure communication application of Sprott A (Nose–Hoover) chaotic flow by using Pecora–Carroll synchronization method (Uyaroglu and Pehlivan 2010). Secure communication of Munmuangsaen–Srisuchinwong five-term chaotic system was implemented with electronic circuit design and passivity based synchronization by Kocamaz et al. (2018). Also, Cicek et al. (2019) implemented the secure communication of five-term jerk chaotic system with electronic circuit design and sliding mode synchronization.

In this study, we have investigated whether there is a chaotic system, which can simply unify some of the five-term 3D chaotic flows and can yield the continued transition from one to another. This paper claims that four especially simple chaotic systems can be combined in this way. In other words, the principal motivation of this work is to develop and analyze a new unified chaotic system, which includes specifically Sprott B, Sprott C, van der Schrier–Mass, and Munmuangsaen–Srisuchinwong autonomous systems. Unified chaotic systems include various chaotic regions. They also have the opportunity to switch one chaotic signal characteristic to another by arranging the values of parameters. Therefore, this simple unified chaotic system has substantially complex behaviors and it is more appropriate for engineering applications.

The rest of this paper is constructed as follows: In the next section, the proposed continuous-time three-dimensional unified system is given. In Sect. 3, the electronic circuit design is demonstrated. Some basic properties such as sensitivity to initial conditions, dissipativity, Lyapunov and Kaplan–Yorke dimensions, bifurcation diagrams, equilibria, and eigenvalues are given in Sect. 4. In the last section, the concluding remarks are presented.

2 Description of Unified Chaotic System

Sprott (1994) explored 19 three-dimensional chaotic flows. The Sprott B chaotic flow is defined as:

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x - y, \\ \dot{z} = 1 - xy, \end{cases} \tag{1}$$

where x, y, z are state variables and the "$\cdot$" denotes the differentiation with respect to t.

The Sprott C chaotic flow is topologically equivalent to Sprott B chaotic flow, but they have distinct structures. It is given in the following differential equations (Sprott 1994):

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x - y, \\ \dot{z} = 1 - x^2. \end{cases} \tag{2}$$

Table 1 Comparison of the new six-term 3D unified chaotic system

Eq. No	Type of chaotic flow	Transformation parameters into a known form of this work			Number of terms	
		a	*b*	*c*	Total	Nonlinear
(1)	Sprott B	1	1	1	5	2
(2)	Sprott C	1	1	0	5	2
(3)	Van der Schrier–Mass	1	$0 < b \leq 1$	1	5	2
(5)	Munmuangsaen–Srisuchinwong	5	90	1	5	2
(8)	This work	Switching possibility between 4 known chaotic flows			6	3

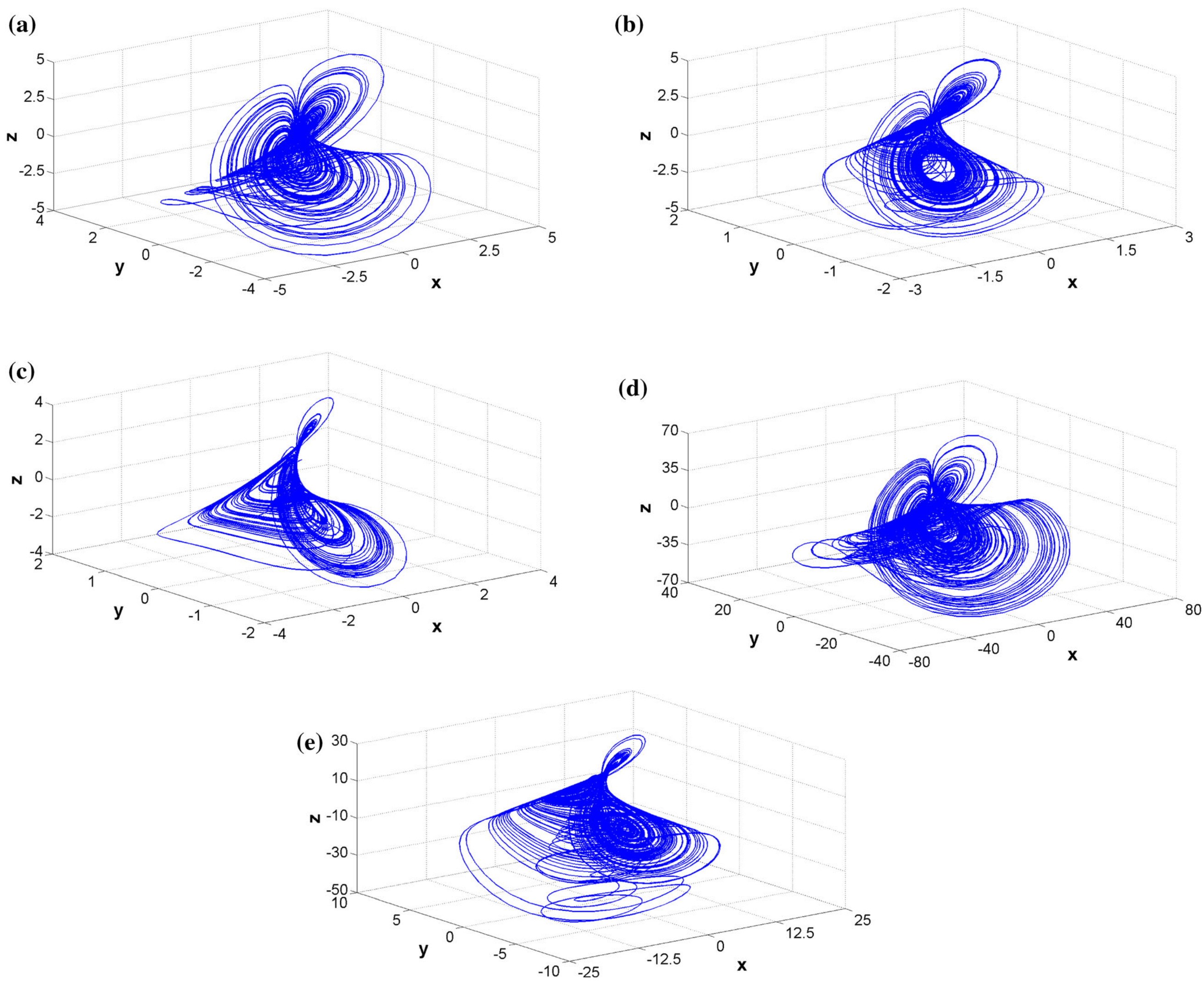

Fig. 1 3D state space plots of unified chaotic system for **a** $a = 1$, $b = 1$, $c = 1$ (Sprott B), **b** $a = 1$, $b = 1$, $c = 0$ (Sprott C), **c** $a = 1$, $b = 0.5$, $c = 1$ (van der Schrier–Mass), **d** $a = 5$, $b = 90$, $c = 1$ (Munmuangsaen–Srisuchinwong), **e** $a = 3$, $b = 25$, $c = 0.5$

Van der Schrier and Maas (2000) introduced a reduced form of Lorenz chaotic system. This system has only five terms, and it is expressed as a set of three first-order, autonomous, ordinary differential equations as follows:

$$\begin{cases} \dot{x} = -y - x, \\ \dot{y} = -xz, \\ \dot{z} = xy + R, \end{cases} \tag{3}$$

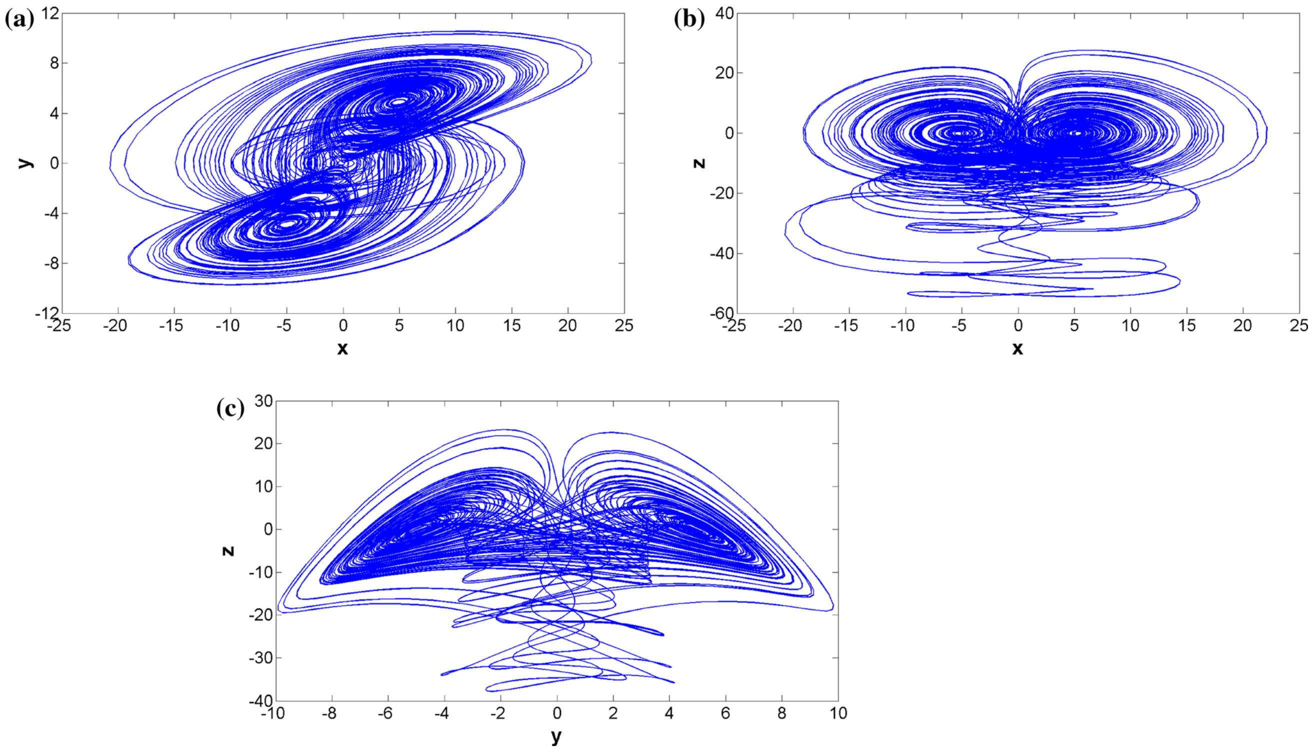

Fig. 2 2D state plots of unified chaotic system with $a = 3$, $b = 25$ and $c = 0.5$ in **a** x–y phase portrait, **b** x–z phase portrait, **c** y–z phase portrait

where $0 < R \leq 1$. When x state is translated to y and y state is translated to $-x$, it is still topologically equivalent and results in the following 3D system:

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x - y, \\ \dot{z} = R - xy. \end{cases} \quad (4)$$

Munmuangsaen and Srisuchinwong (2009) presented a chaotic system with only five terms that consists of two quadratic nonlinearities in three simple differential equations as follows:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = -xz, \\ \dot{z} = -b + xy, \end{cases} \quad (5)$$

where $a = 5$, $b = 90$. When x state is translated to y, y state is translated to x, and z state is translated to $-z$, it is still topologically equivalent and results in the following 3D system:

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = a(x - y), \\ \dot{z} = b - xy. \end{cases} \quad (6)$$

In this study, a novel unified chaotic system is proposed, which is the unity of Sprott B, Sprott C, van der Schrier–Mass, and Munmuangsaen–Srisuchinwong chaotic flows. The differential equations of the unified chaotic system are:

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = a(x - y), \\ \dot{z} = b - cxy - dx^2. \end{cases} \quad (7)$$

where a, b, c and d are system parameters. However, some of the chosen parameters are not independent. One of them is an amplitude parameter as can be seen by making the transformation $x = kx, y = ky$. If $k = 1/\sqrt{c}$, the third equation reduces to $\dot{z} = b - xy - (d/c)x^2$; and if $k = 1/\sqrt{d}$ the third equation reduces to $\dot{z} = b - (c/d)xy - x^2$ without any change in the dynamics including the Lyapunov exponents. A rescaling of x in by a factor of $\sqrt{d}$ should completely remove the dependence on d, and a rescaling of x in by a factor of $\sqrt{c}$ should completely remove the dependence on c. Thus, it is unnecessary to treat them as independent parameters. The usual way to study the transition between two such parameters and systems would have been to take $d = 1 - c$ and write the third equation as $\dot{z} = b - cxy - (1 - c)x^2$ so that the transition occurs over the range of $0 < c < 1$. Then, the differential equations of the unified chaotic system become

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = a(x - y), \\ \dot{z} = b - cxy - (1 - c)x^2. \end{cases} \quad (8)$$

This system has three nonlinearities with six terms. As it can be seen easily in Table 1, in which all the reported

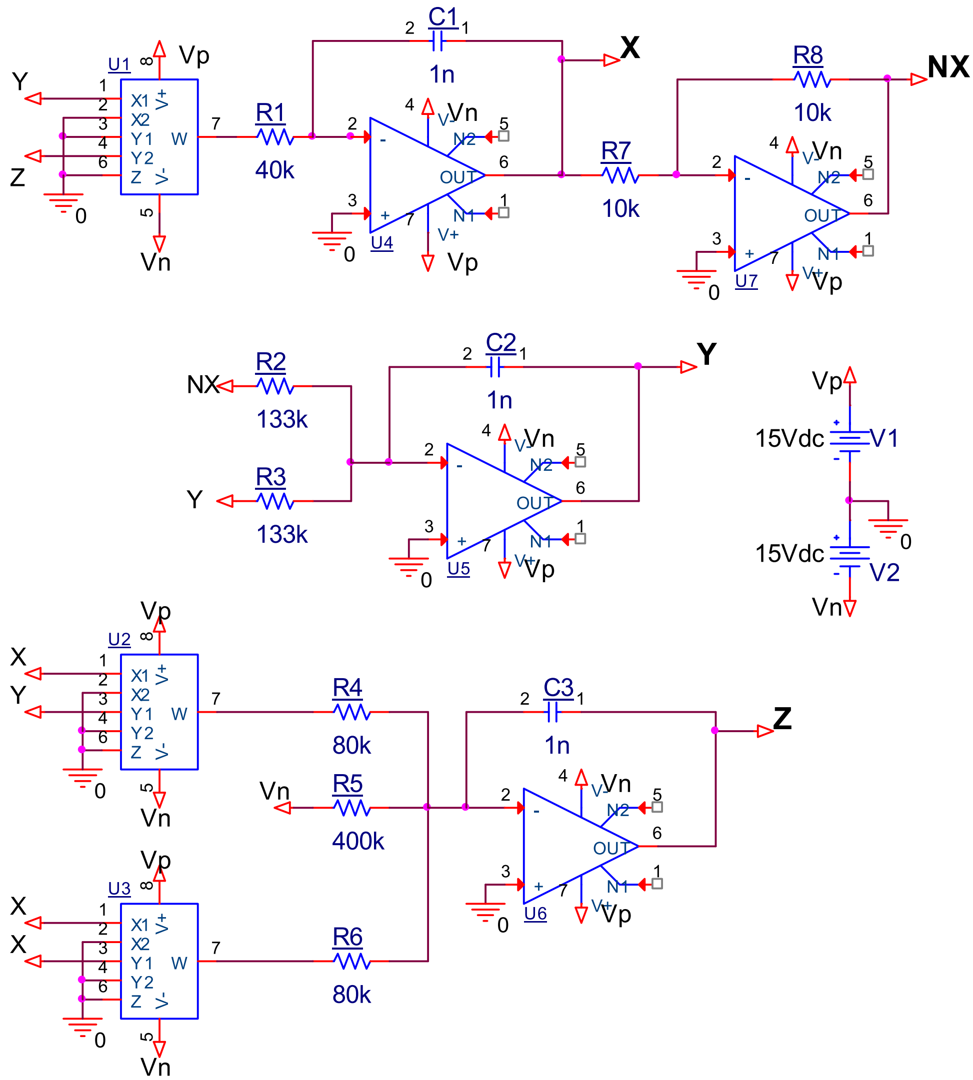

Fig. 3 The circuit design of the unified chaotic system (8)

chaotic systems with "a new 3D unified" are listed and compared, when $a = 1$, $b = 1$ and $c = 1$, system (8) is equivalent to Sprott B chaotic flow; when $a = 1$, $b = 1$ and $c = 0$, it is equivalent to Sprott C chaotic flow; when $a = 1$, $0 < b \leq 1$ and $c = 1$, it is equivalent to van der Schrier–Mass chaotic system; and when $a = 5$, $b = 90$ and $c = 1$, it is equivalent to Munmuangsaen–Srisuchinwong chaotic system.

With the initial conditions $x(0) = 2$, $y(0) = 1.2$, and $z(0) = -0.3$, some 3D state space plots of chaotic system (8) for different parameter values are shown in Fig. 1. The 2D state plots of the unified chaotic system are shown with the parameter values $a = 3$, $b = 25$ and $c = 0.5$ in Fig. 2.

3 Electronic Circuit Design

In this section, we realize theoretical system (8) via an electronic circuit. The circuit is designed with electronics components (see Fig. 3). The circuit includes four operational amplifiers, three capacitors, eight resistors, three analog multipliers.

The values of the components are selected as $R_1 = R_{2\text{-}} = 10$ kΩ, $R_4 = 40$ kΩ, $R_5 = R_6 = 133$ kΩ, $R_7 = 400$ kΩ, $R_8 = R_9 = 80$ kΩ, $C_1 = C_2 = C_3 = 1$ nF. Here, the types of operational amplifiers, analog multipliers are TL084 and AD633, respectively. In Fig. 4, the displayed experimental results of the chaotic dynamics agree with the numerical ones in Fig. 2.

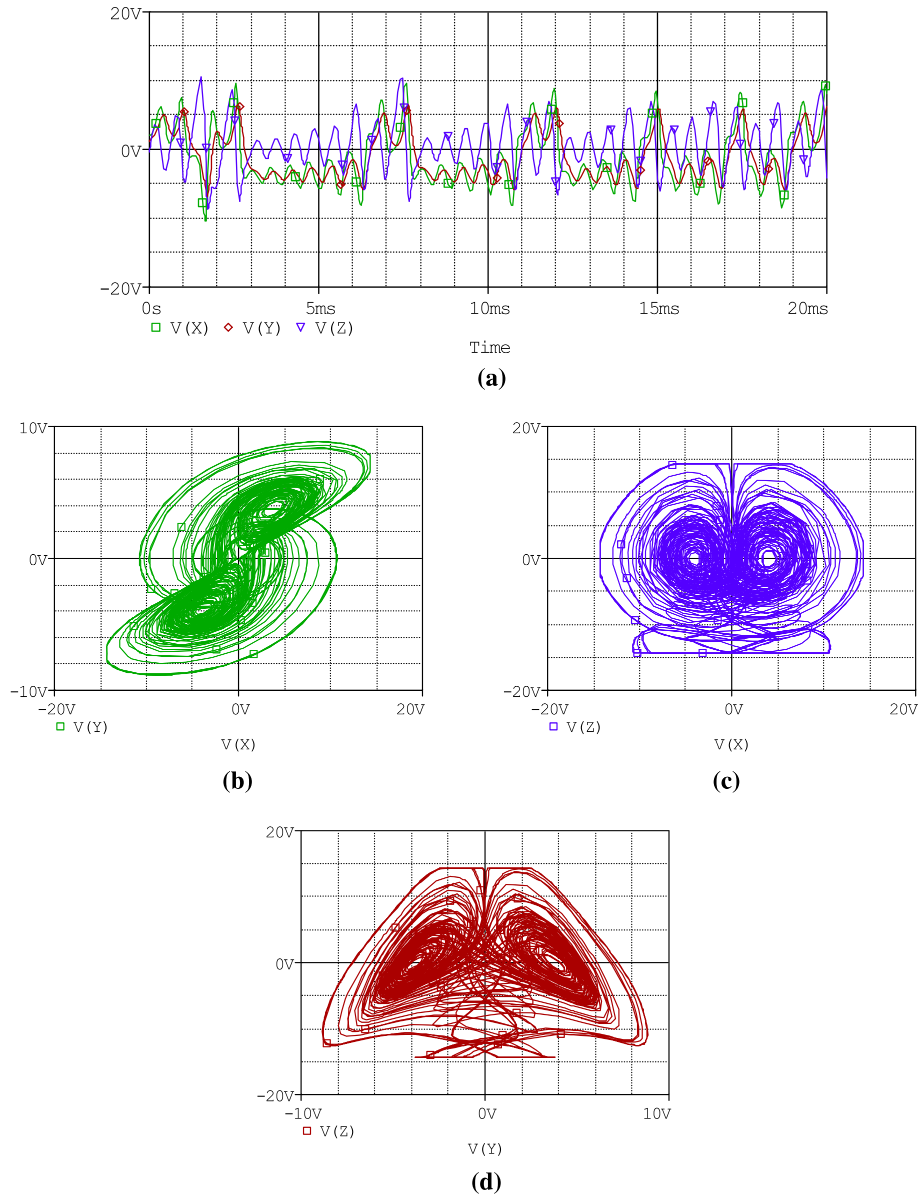

Fig. 4 The electronic circuit outputs of the unified chaotic system **a** time series, **b** x–y phase portrait, **c** x–z phase portrait, **d** y–z phase portrait

4 Basic Properties of Unified Chaotic System

4.1 Sensitivity to Initial Conditions

Sensitivity analysis to initial conditions is one of the noteworthy properties of chaotic systems. The presence of sensitivity to initial conditions in the proposed unified chaotic system can be shown with a simple numerical experiment. When slightly different initial conditions are considered for system (8), for example $y(0) = 1.2$, $y(0) = 1.20001$, and $y(0) = 1.20002$, the sequence becomes very different after a certain time elapsed despite the small differences (see Fig. 5). The same phenomenon is observed for the first and third states of the system even though the initial conditions $x(0) = 2$ and $z(0) = -0.3$ are not changed.

4.2 Dissipativity and the Existence of Attractor

For dynamical system (8), it can be obtained as

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -1. \tag{9}$$

Since $\nabla V = -1$, the dynamical system is a dissipative system and the exponential contraction of system (8) is

$$\frac{\mathrm{d}V}{\mathrm{d}t} = e^{-1}. \tag{10}$$

In the proposed unified chaotic system, a volume element V_0 is obviously impacted by the flow into a volume

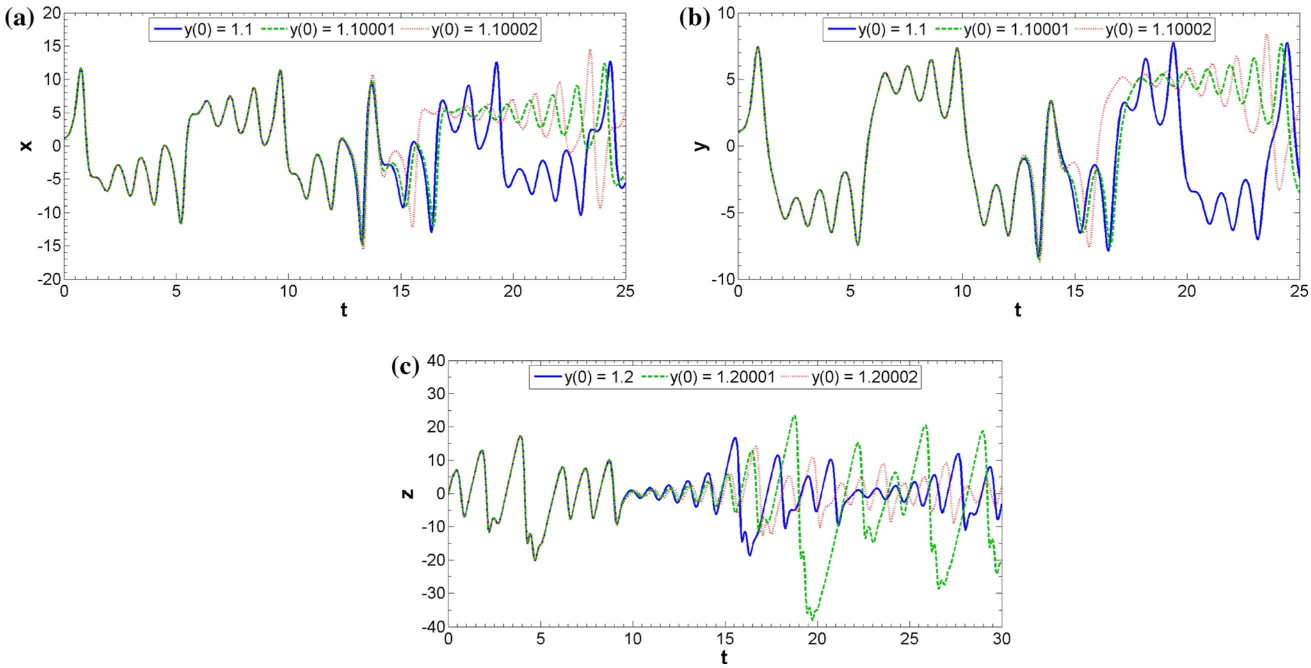

Fig. 5 Time series of unified chaotic flow with a = 3, b = 25 and c = 0.5 for **a** x signals, **b** y signals, **c** z signals

element V_0e^{-t} at time t, which means each volume comprising the trajectory of this dynamical system shrivels to zero as time goes to infinity at an exponential rate $-$ 1. Thus, all orbits of this dynamical system are finally bounded to a specific subset, which has zero volume, the asymptotic motion arranges into an attractor of system (8).

4.3 Lyapunov Exponents

Lyapunov exponents represent a mathematical and numerical means of probing a system for chaotic or stable behavior, which characterizes the rating separation of infinitesimally close trajectories of a dynamical system. Positive maximal Lyapunov exponent is one of the key components of chaotic dynamics. Lyapunov exponents measure the average exponential rating divergence or convergence of nearby trajectories in the state space for a dynamical system. If a system has all negative Lyapunov exponents, this system is a fixed point. If a system has one zero Lyapunov exponent with all rest negative exponents, the motion of the system is a limit cycle. If a system has at least one positive Lyapunov exponents with one zero and one negative exponents, then it is defined as chaotic by Chen and Ueta (1999).

The Lyapunov exponents of the proposed unified chaotic system are demonstrated for different parameter values in Fig. 6a, c and e. When parameters are a = 3, b = 25 and c = 0.5, the Lyapunov exponents of system (8) are calculated by using the algorithm of Wolf et al. (1985) as follows:

$$L_1 = 0.9, L_2 = 0 \quad \text{and} \quad L_3 = -3.89. \tag{11}$$

So that one of the Lyapunov exponents is positive, system (8) possesses expanding nature of different directions in phase space, the proposed unified system is chaotic. The geometry and the density of chaotic systems can be difficult to describe. Some quantitative characterizations such as Lyapunov and Kaplan–Yorke dimensions of the attractors are used. The Lyapunov dimension of system (8) is fractionally described by

$$D_L = 1 - \frac{L_1}{L_3} = 1 - \frac{0.9}{-3.89} = 1.231. \tag{12}$$

where L_l> L_3. Kaplan–Yorke dimension is a useful tool for determining the fractal dimension and the rate of entropy production of the considered dynamical attractor. For a three-dimensional autonomous system, the Kaplan–Yorke dimension is between 2.0 and 3.0. It is generally slightly greater than 2.0. The Kaplan–Yorke dimension of proposed unified chaotic system is calculated as:

$$D_{\mathrm{KY}} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 2 + \frac{0.9 + 0}{|-3.89|} = 2.231. \tag{13}$$

4.4 Bifurcation Diagrams

A bifurcation diagram is a graphical depiction of the relationship between the values of one parameter and the behavior of a nonlinear system in which the parameter is being measured. In other words, it shows the sudden

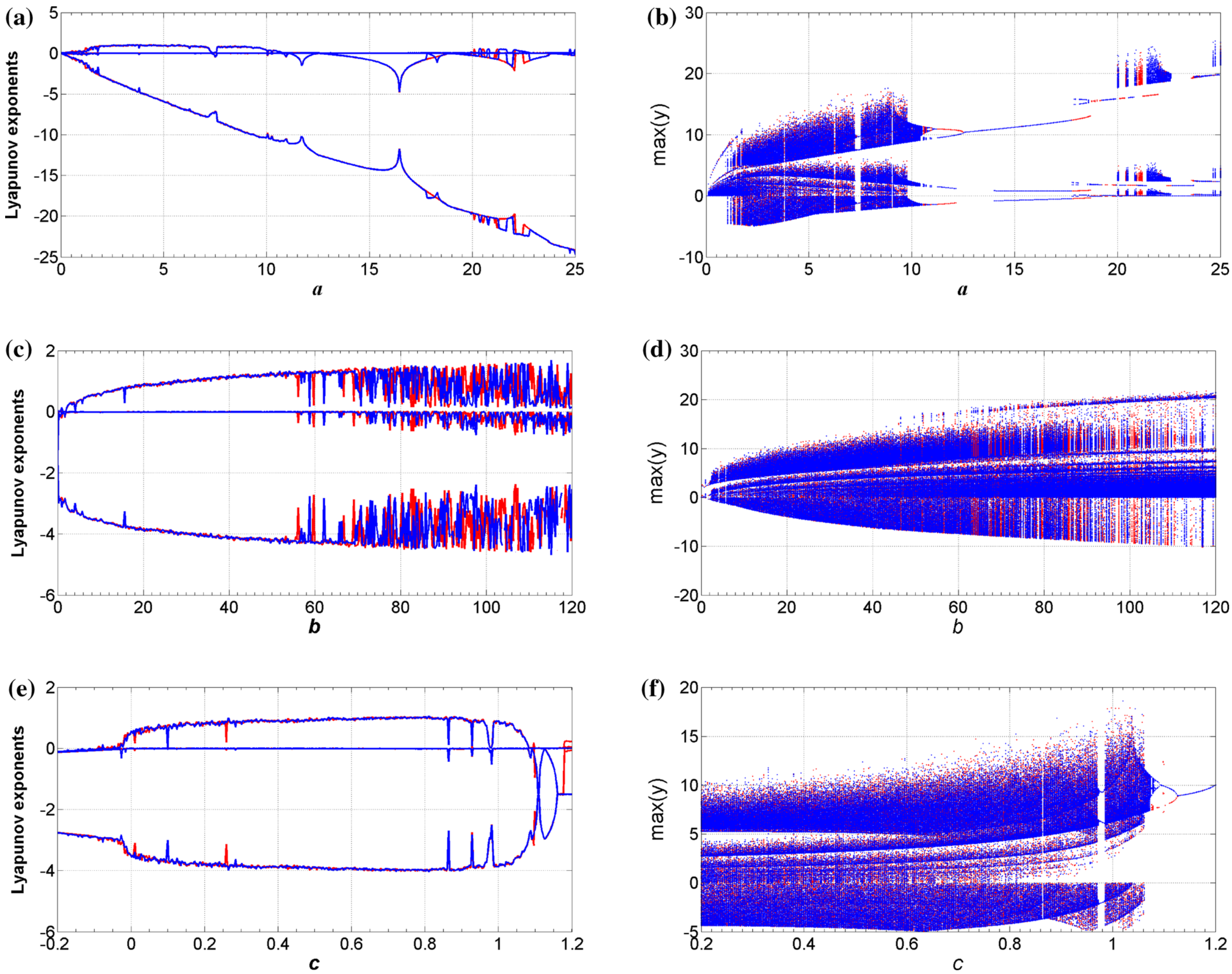

Fig. 6 Lyapunov exponents (left) and bifurcation maps (right) with blue color for initial conditions (2, 1.2, − 0.3) and red color for initial conditions (0.5, 0.5, 0.5) for **a**, **b** a parameter with $b = 25$ and $c = 0.5$, **c**, **d** b parameter $a = 3$ and $c = 0.5$, **e**, **f** c parameter $a = 3$ and $b = 25$

appearance of qualitatively different solutions such as fixed points, periodic orbits or chaotic attractors for a dynamical system as a function of a parameter is varied. The extremum values of one of the state variables are considered. The stable values are usually represented with a solid line, and the unstable values are represented with a dotted line, though the unstable points are often omitted. The bifurcation of unified system (8) is demonstrated for varying a in Fig. 6b, for varying b in Fig. 6d, and for varying c in Fig. 6f. For displaying of the figures, we used blue color for initial conditions (2, 1.2, − 0.3) and red color for initial conditions (0.5, 0.5, 0.5).

The bifurcation maps in Fig. 6 show that there are some gaps, in which the proposed unified chaotic system has limit cycles. The local Lyapunov exponents and bifurcation analysis for varying c in 0.96 and 0.99 with $a = 3$ and $b = 25$; x time series and 3D state space plot of unified chaotic system (8) with parameter values $a = 3$, $b = 25$ and $c = 0.98$ are given in Fig. 7.

4.5 Nontrivial Equilibria, Jacobian Matrix, Eigenvalues

The equilibria of proposed unified chaotic system can be calculated by obtaining $\dot{x} = 0, \dot{y} = 0, \dot{z} = 0$, with the solution of the following system:

$$\begin{aligned} & yz = 0, \\ & a(x - y) = 0, \\ & b - cxy - (1 - c)x^2 = 0. \end{aligned} \tag{14}$$

So, it has two equilibrium points: $E_1(\sqrt{b}, \sqrt{b}, 0)$ and $E_2(-\sqrt{b}, -\sqrt{b}, 0)$. This system has nontrivial equilibria which means no zero equilibria.

The Jacobian matrix of system (8) is

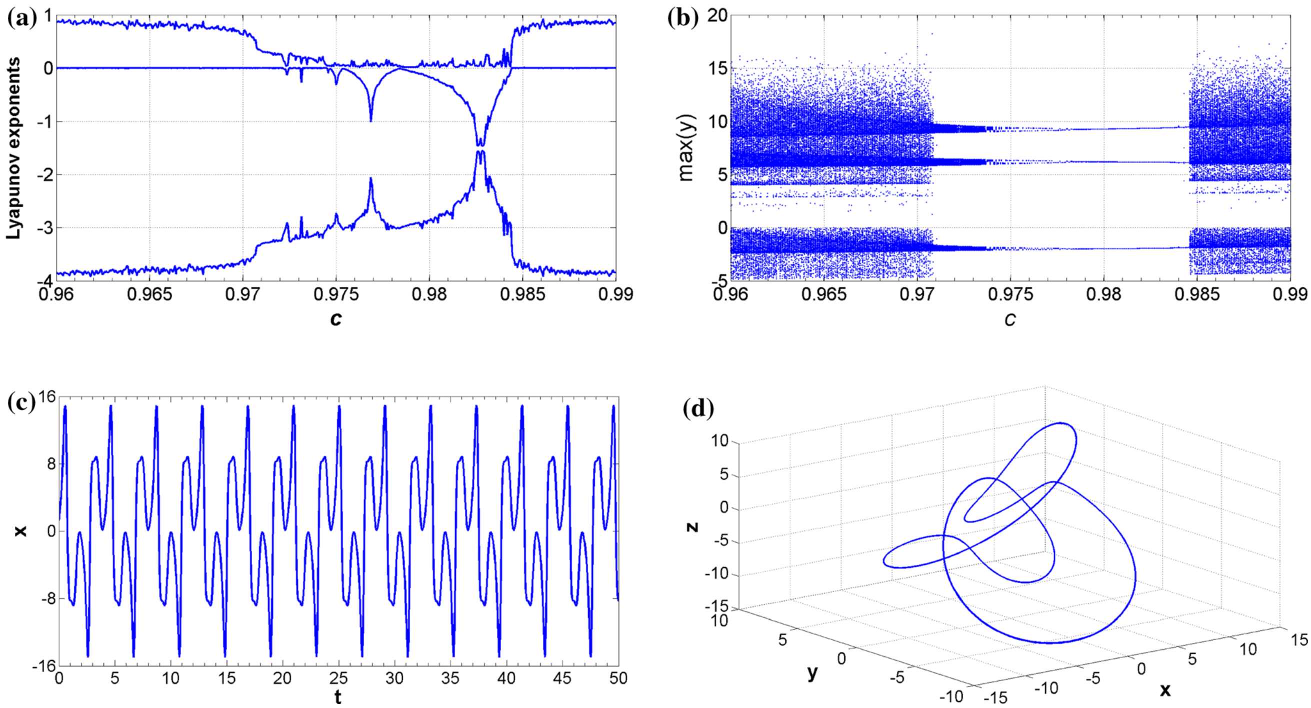

Fig. 7 The unified chaotic system for $a = 3$, $b = 25$, **a** Lyapunov exponents for varying c in 0.96 and 0.99, **b** bifurcation map, **c** x time series for $c = 0.98$, **d** 3D state space plot for $c = 0.98$

$$J = \begin{pmatrix} 0 & z & y \\ a & -a & 0 \\ -cy - 2(1-c)x & -cx & 0 \end{pmatrix}. \tag{15}$$

The corresponding Jacobian matrices for $E_1(\sqrt{b}, \sqrt{b}, 0)$ and $E_2(-\sqrt{b}, -\sqrt{b}, 0)$ are:

$$J(E_1) = \begin{pmatrix} 0 & 0 & \sqrt{b} \\ a & -a & 0 \\ (c-2)\sqrt{b} & -c\sqrt{b} & 0 \end{pmatrix}, \tag{16}$$

and

$$J(E_2) = \begin{pmatrix} 0 & 0 & -\sqrt{b} \\ a & -a & 0 \\ -(c-2)\sqrt{b} & c\sqrt{b} & 0 \end{pmatrix}. \tag{17}$$

To calculate the eigenvalues, let $|\lambda I - J(E_1)| = 0$ and $|\lambda I - J(E_2)| = 0$; then, the same characteristic polynomial is obtained for the nontrivial equilibrium points E_1 and E_2 as follows:

$$\lambda^3 + a\lambda^2 + (2-c)b\lambda + 2ab = 0. \tag{18}$$

So, they give the same eigenvalues. For $a = 3$, $b = 25$ and $c = 0.5$, they are calculated as:

$$\lambda_1 = -3.7294, \lambda_2 = 0.3647 + 6.3315i, \\ \lambda_3 = 0.3647 - 6.3315i. \tag{19}$$

Finally, $\lambda_1 \in \mathbb{R}^-$, λ_2 and $\lambda_3 \in \mathbb{C}$, which are conjugate with positive real parts. That means the nontrivial equilibriums E_1 and E_2 are unstable saddle-focus points.

4.6 Switching to Other Chaotic Flows

Transition of the proposed unified system to Sprott B, Sprott C, van der Schrier–Mass, and Munmuangsaen–Srisuchinwong chaotic flows is examined. As shown in Fig. 8, unified system (8) is started with parameters $a = 3$, $b = 25$, $c = 0.5$, then their values are changed when $t > 100$, and switching to these chaotic flows is achieved successfully. Therefore, this simple unified chaotic system includes more complex dynamical behaviors. Switching to other chaotic flows makes it more proper for engineering applications such as image encryption, secure communication and random number generator. However, the values of states at the cutoff point may cause some problems because of the sensitive dependence on initial values of chaotic systems. For this reason, synchronization with unknown parameters can be needed. When the synchronization rules are applied to the chaotic systems, two different signals are synchronized. Based on the control and stability theory, the synchronization of chaotic systems has been designed with unknown parameters by using some effective control methods (Zhang et al. 2015; Driss and Mansouri 2016; Gao et al. 2016; Sun et al. 2017; Tirandaz and Saeidiaminabadi 2017; Tirandaz et al. 2018).

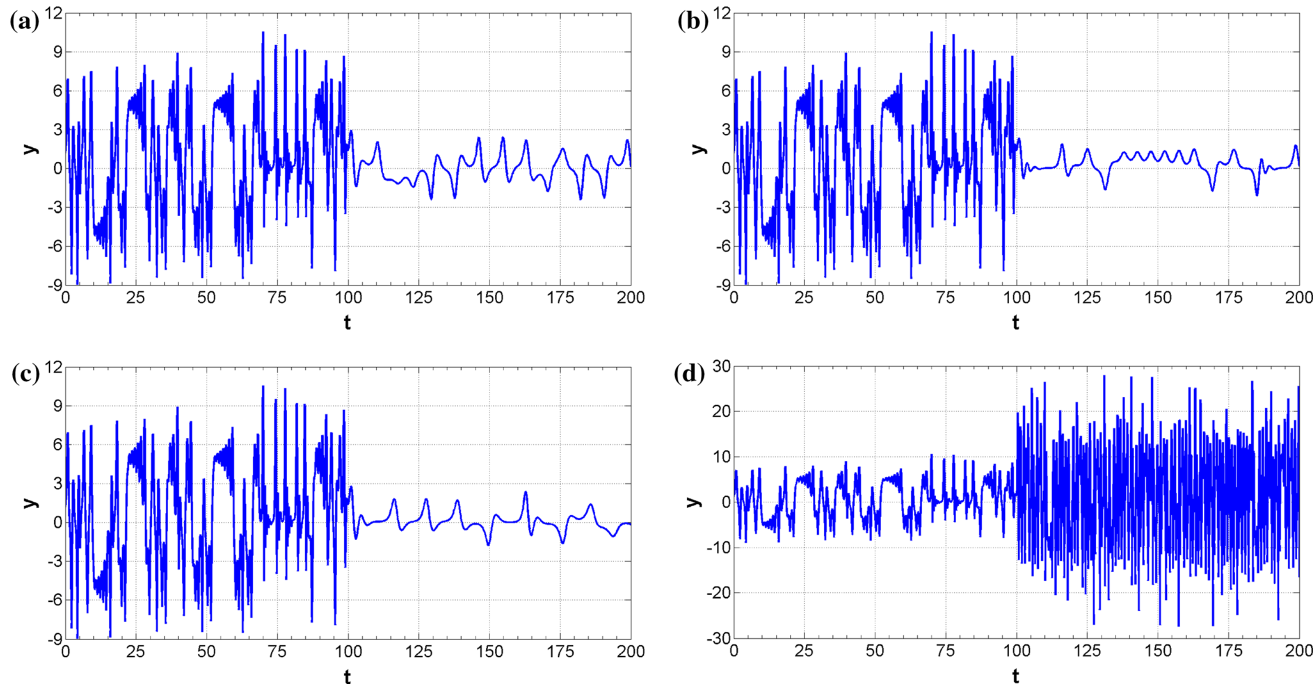

Fig. 8 The y state of unified chaotic system starting with parameters a = 3, b = 25, c = 0.5, and when t > 100 switching to **a** a = 1, b = 1, c = 1 (Sprott B), **b** a = 1, b = 1, c = 0 (Sprott C), **c** a = 1, b = 0.5, c = 1 (van der Schrier–Mass), **d** a = 5, b = 90, c = 1 (Munmuangsaen–Srisuchinwong)

5 Conclusion

This paper introduces a new unified chaotic system, which can identify with a kind of unique and unified classification of four chaotic attractors containing the Sprott B, Sprott C, Van der Schrier–Mass, and Munmuangsaen–Srisuchinwong chaotic flows. This unified chaotic system has substantially complex dynamical behaviors. In addition, this unified chaotic system has:

- Become the smallest unified chaotic system. It consists of six terms, three of which have nonlinearities,
- Contributed to a better understanding of the relationship between Sprott B, Sprott C, van der Schrier–Mass, and Munmuangsaen–Srisuchinwong chaotic systems,
- Moreover, including four different chaotic flows is an advantage for the engineering applications such as chaotic mixer, medical electronics, and secure communication,
- For ease of use in engineering, such compact unified chaotic attractor models with new combinations must be further explored and examined.

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Compliance with Ethical Standards

Conflicts of Interest The authors declare that they have no conflict of interest.

Human and Animal Rights All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards. For this type of study, formal consent is not required.

Informed Consent Informed consent was obtained from all individual participants included in the study.

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