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ORIGINAL RESEARCH

Padovan numbers as difference of two repdigits

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Abstract In this paper, we find all Padovan numbers which can be written as are difference of two repdigits. It is shown that all Padovan numbers which can be written as a difference of two repdigits are $P_k \in \{2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 200, 3329\}$.

Keywords Diophantine equations · Continued fraction · Repdigit · Linear forms in logarithms · Padovan number

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1 Introduction

Positive integers with all digits equal are called repdigit. The number of repdigits for many special sequences has been searched. Below are the repdigit numbers found for some sequences:

$$\{0, 1, 2, 3, 5, 8, 55\} \text{ for Fibonacci numbers [1]},$$
$$\{1, 2, 3, 4, 7, 11\} \text{ for Lucas numbers [1]},$$
$$\{0, 1, 3, 7\} \text{ for Pell numbers [2]},$$
$$\{2, 6\} \text{ for Pell-Lucas numbers [2]},$$
$$\{0, 1, 2, 3, 4, 5, 7, 9\} \text{ for Padovan numbers [3]},$$

and

$$\{0, 2, 3, 5, 7, 22\} \text{ for Perrin numbers [3]}.$$

In later years, some authors investigated the Fibonacci and Lucas numbers, which are the sum of two repdigits. In [4], it was shown that the largest of this type of Fibonacci number is $F_{20} = 6765 = 6666+99$ by Díaz Alvarado et al.. Similarly, in [5], it was shown that the largest of this type of Lucas number is $L_{14} = 843 = 777 + 66$ by Adegbindin et al.. Later, some special number sequences that can be written as a concatenations of two repdigits were investigated by several authors. In [6], Rayaguru and Panda examined that the balancing number in this form is 35. In [7], the Alahmadi et al. showed that Fibonacci numbers in this form are only 13, 21, 34, 55, 89, 144, 233 and 377. In [8–11], Keskin et al. worked on the problems of finding Fibonacci or Lucas numbers which are product,

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sum, difference, or concatenations of two or three repdigits. Moreover, in [12–15], Keskin et al. determined repdigits in base b or 10 which are products or sums of two Fibonacci or Lucas numbers. Specially, in [8] and [10], Erduvan and Keskin showed that all Lucas numbers which are concatenations of two repdigits are only 18, 29, 47, 76, 199, 322 and concatenations of three repdigits are only 123, 199, 322, 521, 843, 2207, 5778. In [16], Batte et al. showed that all Perrin numbers which are concatenations of two distinct repdigits are $P_n \in \{10, 12, 17, 29, 39, 51, 68, 90, 119, 277, 644\}$. In [17], Chalebgwa and Ddamulira showed that the only Padovan numbers which are palindromic concatenations of two distinct repdigits are $P_n \in \{151, 616\}$. In [24], Ddamulira explored all repdigits as sums of three Padovan numbers. In [18], Bhoi and Ray investigated Perrin numbers, which can be represented as sums of two base b repdigits. In [19], Rihane and Togbé studied repdigits which are products of consecutive Padovan or Perrin numbers and in [20], they manifested Padovan numbers and Perrin numbers which are products of two repdigits. In [21], Lomelí and Hernández determined repdigits which are sums of two Padovan numbers.

Let η be an algebraic number of degree d with minimal polynomial

$$a_0 \prod_{i=1}^{d} \left(x - \eta^{(i)}\right) \in \mathbb{Z}[x],$$

where the $\eta^{(i)}$'s are conjugates of η and the a_i's are relatively prime integers with $a_0 > 0$. Then, $h(\eta)$, the logarithmic height of η, is

$$\frac{1}{d}\left(\log a_0 + \sum_{i=1}^{d} \log\left(\max\left\{|\eta^{(i)}|, 1\right\}\right)\right). \tag{1}$$

Moreover, if $\eta = a/b \in \mathbb{Q}$, $\gcd(a, b) = 1$ and $b \geq 1$, then $h(\eta) = \log(\max\{|a|, b\})$.

Proof of the following properties is found in [22].

$$h(\eta) + h(\gamma) \geq h(\eta\gamma^{\pm 1}), \tag{2}$$
$$\log 2 + h(\eta) + h(\gamma) \geq h(\eta \pm \gamma), \tag{3}$$
$$h(\eta^m) = |m| h(\eta). \tag{4}$$

Let $(P_k)_{k\geq 0}$ be the sequence of Padovan numbers given by

$$P_0 = 0,\ P_1 = P_2 = 1,\ P_k = P_{k-2} + P_{k-3}$$

for $k \geq 3$.

$$\alpha = \frac{\sqrt[3]{108 + 12\sqrt{69}} + \sqrt[3]{108 - 12\sqrt{69}}}{6},$$
$$\beta = \frac{-(\sqrt[3]{108 + 12\sqrt{69}} + \sqrt[3]{108 - 12\sqrt{69}}) + i\sqrt{3}(\sqrt[3]{108 + 12\sqrt{69}} - \sqrt[3]{108 - 12\sqrt{69}})}{12} = \overline{\gamma}.$$

are the roots of the characteristic equation $x^3 - x - 1 = 0$.

The Binet formula for the Padovan numbers is

$$P_k = t \cdot \alpha^k + s \cdot \beta^k + r \cdot \gamma^k$$

where

$$t = \frac{\alpha(\alpha+1)}{2\alpha+3},\ s = \frac{\beta(\beta+1)}{2\beta+3},\ r = \frac{\gamma(\gamma+1)}{2\gamma+3}.$$

Then it is known that

$$\alpha^{k-3} \leq P_k \leq \alpha^{k-1}, \text{ for } k \geq 1,$$
$$1.32 < \alpha < 1.33,$$
$$0.86 < |\beta| = |\gamma| = \alpha^{-1/2} < 0.87,$$
$$h(t) \leq \frac{1}{3}\log 23,$$

and

$$0.28 < |s| = |r| < 0.29.$$

Moreover, for $k \geq 1$, it can known that

$$|e(k)| := |r \cdot \gamma^k + s \cdot \beta^k| \leq |r| \cdot |\gamma|^k + |s| \cdot |\beta|^k = \alpha^{-k/2}|s| + \alpha^{-k/2}|r| < \frac{1}{\alpha^{k/2}}.$$

In addition, the minimal polynomial of t over $\mathbb{Z}$ is given by $23x^3 - 5x - 1$ and zeros of this equation are r, s, t. See [3,23–25], etc., for more information on Padovan sequences.

Let $F := \mathbb{Q}(\alpha, \beta)$ be the splitting field of the polynomial ϕ over $\mathbb{Q}$. Then, we have $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 3$, $|Gal(F/\mathbb{Q})| = [F : \mathbb{Q}] = 6$ and

$$Gal(F/\mathbb{Q}) \simeq \{(1), (\beta\gamma), (\alpha\beta), (\alpha\gamma), (\alpha\beta\gamma), (\alpha\gamma\beta)\} \simeq S_3.$$

We will use it for the permutation $(\alpha\beta)$.

In this article, the Padovan numbers, which are the difference of two repdigits, were examined and found to be $P_k \in \{2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 200, 3329\}$. To find these solutions, the Diophantine equation

$$P_k = \frac{d_1(10^n - 1)}{9} - \frac{d_2(10^m - 1)}{9} \tag{5}$$

is solved where $n \geq 2$, k and m are positive integers. Also, it was concluded that the equation $P_k = 10^n - 10^m$ has no solutions. We will apply Baker's theory of lower bounds to solve this equation.

2 Preliminaries

We will give the four lemmas necessary to solve our problem. These lemmas are popular lemmas that are often used in similar studies. We will reduce the k sub-index with the help of these lemmas.

Lemma 1 is deduced from Corollary 2.3 of Matveev [26] (also see Theorem 9.4 in [27]).

Lemma 1 *Let $b_1, b_2 \ldots, b_t$ be nonzero integers, $\gamma_1, \gamma_2, \ldots, \gamma_t$ be positive real algebraic numbers in a real algebraic number field $\mathbb{K}$ of degree D, and*

$$\Lambda := \gamma_1^{b_1} \cdots \gamma_t^{b_t} - 1 \neq 0$$

Then, for all $i = 1, 2 \ldots, t$,

$$\exp\left(-30^{t+3} t^{4.5} (1 + \log D) 1.4 D^2 A_1 A_2 \cdots A_t (1 + \log B)\right) < |\Lambda|,$$

where

$$\max\{|b_1|, |b_2| \ldots, |b_t|\} \leq B,$$

and $\max\{0.16, Dh(\gamma_i), |\log \gamma_i|\} \leq A_i$.

Now we will now give the lemma proven in [28], which is a different type of the lemma given by Dujella and Pethő in [29].

Lemma 2 *Let the function $||\cdot||$ denote the distance from x to the nearest integer. Let $A > 0$, μ, $B > 1$ be some real numbers u, v, w, M be positive integers, and p/q be a convergent of the continued fraction of the irrational number γ where $q > 6M$. Let $\epsilon := ||\mu q|| - M||\gamma q||$. If $\epsilon > 0$, then there exists no solution to the inequality*

$$0 < |u\gamma - v + \mu| < AB^{-w},$$

with

$$\frac{\log(Aq/\epsilon)}{\log B} \leq w \text{ and } u \leq M.$$

Lemma 3 [31] *Assume that $a, x \in \mathbb{R}$. If $|x| < a$ and $0 < a < 1$, then*

$$|\log(x+1)| < \frac{\log(1/(1-a))}{a} \cdot |x|$$

and

$$|x| < \frac{-a}{e^{-a}-1} \cdot \left|e^{x}-1\right|.$$

The following lemmas are given in [3].

Lemma 4 *The largest Padovan numbers that can be written as a repdigit is $P_{11} = 9$.*

3 Main Theorem

In this section, we will give a theorem about Padovan numbers and we will use Matlab for all our calculations.

Theorem 5 *If P_k is expressible as difference of two repdigits, then*

$$P_k \in \{2, 3, 4, 5, 7, 9, 12, 16, 21, 28, 37, 49, 65, 86, 200, 3329\}.$$

Proof Suppose that the equation (5) is valid. We will solve this equation in three cases.

Case 1: Suppose $1 \leq k \leq 573$ and $n \geq 2$. It can be shown that

$$\begin{aligned}
P_4 &= P_5 = 2 = 11-9,\ P_6 = 3 = 11-8,\\
P_7 &= 4 = 11-7,\ P_8 = 5 = 11-6,\\
P_9 &= 7 = 11-4,\ P_{10} = 9 = 11-2,\\
P_{11} &= 12 = 111-99,\ P_{12} = 16 = 22-6,\\
P_{13} &= 21 = 22-1,\ P_{14} = 28 = 33-5,\\
P_{15} &= 37 = 44-7,\ P_{16} = 49 = 55-6,\\
P_{17} &= 65 = 66-1,\ P_{18} = 86 = 88-2,\\
P_{21} &= 200 = 222-22,\ P_{31} = 3329 = 3333-4
\end{aligned}$$

by using MATLAB.

Case 2: Suppose $k \geq 573$ and $n = m$. It can be easily shown that $d_1 > d_2$ and P_k is a repdigit. This is impossible by Lemma 4.

Case 3: Suppose $k \geq 573$ and $n - m \geq 1$. By (5), we write

$$P_k = t \cdot \alpha^k + s \cdot \beta^k + r \cdot \gamma^k = \frac{d_1 \cdot (10^n - 1)}{9} - \frac{d_2 \cdot (10^m - 1)}{9}.$$

We will use this equation by arranging it in two different ways as follows.

$$9 \cdot t \cdot \alpha^k - d_1 \cdot 10^n = -9(s \cdot \beta^k + r \cdot \gamma^k) - d_2 \cdot 10^m - (d_1 - d_2), \tag{6}$$

and

$$t \cdot \alpha^k - \frac{d_1 \cdot 10^n - d_2 \cdot 10^m}{9} = -(s \cdot \beta^k + r \cdot \gamma^k) - \frac{(d_1 - d_2)}{9}. \tag{7}$$

Let's take absolute value of these equation and re-arrange them as follows. Then, we write

$$\begin{aligned}
\left|\frac{9 \cdot t \cdot \alpha^k}{d_1 \cdot 10^n} - \frac{d_1 \cdot 10^n}{d_1 \cdot 10^n}\right| &= \left|\frac{9 \cdot 10^{-n} \cdot t \cdot \alpha^k}{d_1} - 1\right| \\
&\leq \frac{9|s \cdot \beta^k + r \cdot \gamma^k|}{d_1 10^n} + \frac{d_2 10^m}{d_1 10^n} + \frac{|d_1 - d_2|}{d_1 10^n} \\
&\leq \frac{9(|e(k)|}{d_1 10^n} + \frac{d_2}{d_1 10^{n-m}} + \frac{8}{d_1 10^n}
\end{aligned}$$

$$
\begin{aligned}
&\leq \frac{9\alpha^{-k/2}}{d_1 10^n} + \frac{d_2}{d_1 10^{n-m}} + \frac{8}{d_1 10^n} \\
&\leq \frac{9\alpha^{-k/2}}{10^{n-m+1}} + \frac{9}{10^{n-m}} + \frac{8}{10^{n-m+1}} \\
&< \frac{9.81}{10^{n-m}}. \qquad (8)
\end{aligned}
$$

and

$$
\begin{aligned}
\left|\frac{t\cdot\alpha^k}{t\cdot\alpha^k} - \frac{d_1 10^n - d_2 10^m}{9\cdot t\cdot\alpha^k}\right| &= \left|1 - \frac{(d_1 - d_2 10^{m-n})\cdot 10^n\cdot\alpha^{-k}}{9t}\right| \\
&\leq \left|\frac{(s\cdot\beta^k + r\cdot\gamma^k)}{t\cdot\alpha^k}\right| + \left|\frac{(d_1-d_2)}{9\cdot t\cdot\alpha^k}\right| \\
&\leq \left|\frac{e(k)}{t\cdot\alpha^k}\right| + \frac{|d_1-d_2|}{9\cdot t\cdot\alpha^k} \\
&\leq \frac{\alpha^{-k/2}}{t\cdot\alpha^k} + \frac{8}{9\cdot t\cdot\alpha^k} < \frac{1.65}{\alpha^k}, \qquad (9)
\end{aligned}
$$

To apply Lemma 1, we take

$$
(\Lambda_1, \gamma_1, \gamma_2, \gamma_3, b_1, b_2, b_3) := \left(\frac{\alpha^k\cdot 10^{-n}\cdot 9\cdot t}{d_1} - 1, \alpha, 10, \frac{9\cdot t}{d_1}, k, -n, 1\right) \qquad (10)
$$

and

$$
\begin{aligned}
&(\Lambda_2, \gamma_1', \gamma_2', \gamma_3') := \left(1 - \frac{\alpha^{-k}\cdot 10^n\cdot(d_1 - d_2 10^{m-n})}{9t}, \alpha,\ 10, \frac{(d_1 - d_2 10^{m-n})}{9t})\right) \\
&(b_1', b_2', b_3') := (-k, n, 1). \qquad (11)
\end{aligned}
$$

Moreover, $\mathbb{K} = \mathbb{Q}(\gamma_1, \gamma_2, \gamma_3) = \mathbb{Q}(\alpha)$ or $\mathbb{K} = \mathbb{Q}(\gamma_1', \gamma_2', \gamma_3') = \mathbb{Q}(\alpha)$. Thence, $D = 3$. Additionally, if $\Lambda_1 = 0$, then $t\cdot\alpha^k = \frac{10^n d_1}{9}$ and if $\Lambda_2 = 0$, then $t\cdot\alpha^k = \frac{10^n(d_1 - d_2 10^{m-n})}{9}$. We take an automorphism σ from both sides of these equation and apply absolute values, then it can be shown that

$$
\left|\frac{10^n d_1}{9}\right| = |\sigma(t\alpha^k)| = |s\beta^k| < 1
$$

and

$$
\left|\frac{10^n(d_1 - d_2 10^{m-n})}{9}\right| = |\sigma(t\alpha^k)| = |s\beta^k| < 1,
$$

which are impossible can be shown by a simple calculation. As a result, $\Lambda_1 \neq 0$ and $\Lambda_2 \neq 0$. The logarithmic height for $\gamma_1, \gamma_2, \gamma_3, \gamma_1', \gamma_2', \gamma_3'$ are as follows:

$$
\begin{aligned}
&h(\alpha) := h(\gamma_1) := h(\gamma_1') = \frac{\log\alpha}{3}, \\
&h(\gamma_2) := h(\gamma_2') = \log 10, \\
&h(\gamma_3) := h\left(\frac{t\cdot 9}{d_1}\right) \leq h(t) + h(9) + h(d_1) \leq \frac{1}{3}\log 23 + \log 9 + \log 9 < 5.44, \\
&h(\gamma_3') := h\left(\frac{d_1 - d_2 10^{m-n}}{9t}\right) \\
&\qquad \leq \log 2 + h(9) + h(t) + h(d_1) + h(d_2) + h(10)(n-m) \\
&\qquad \leq (n-m)\log 10 + 8.34.
\end{aligned}
$$

Then, we choose

$$(A_1, A_2, A_3, A_1', A_2', A_3') := (\log\alpha, \log 10^3, 16.32, \log\alpha, \log 10^3, 25.02 + 3\cdot(n-m)\cdot\log 10). \tag{12}$$

Moreover, the inequality

$$\alpha^{8(n-2)} < 10^{n-2} \le 10^{n-2} + 10^{n-1} - 10^{n-1} < \frac{d_1(10^n-1)}{9} - \frac{d_2(10^m-1)}{9} = P_k \le \alpha^{k-1}$$

can be written. Therefore, $n < 8n < k + 15$. Also, since $B \ge \max\{|k|, |n|, 1\}$, we choose

$$B := k + 15. \tag{13}$$

By using Lemma 1, (8), (10), (12), and (13), it can be written that

$$\begin{aligned} &10^{-n+m}\cdot(9.81) > |\Lambda_1| \\ &\quad > \exp\left(-30^6\cdot(1.4)\cdot 3^{4.5}\cdot 3^2\cdot(1+\log 3)\cdot\log\alpha\cdot\log 10^3\cdot 16.32\cdot(1+\log(k+15))\right), \end{aligned}$$

i.e.,

$$(m-n)\log 10 > -8.58\cdot 10^{13}\cdot(1+\log(k+15)) - \log(9.81). \tag{14}$$

Similarly, by using (9), (11), (12), (13), and Lemma 1, we have

$$\begin{aligned} 1.65\cdot\alpha^{-k} &> |\Lambda_2| \\ &> \exp\Big(-1.4\cdot 30^6\cdot 3^{6.5}(1+\log 3)\cdot\log\alpha\cdot\log 10^3\cdot(1+\log(k+15)) \\ &\qquad \cdot\left(25.02 + \log 10^3\cdot(n-m)\right)\Big) \end{aligned}$$

i.e.,

$$-k\log\alpha + \log(1.65) > -5.26\cdot 10^{12}\cdot(1+\log(k+15))\,(25.02 + 3(n-m)\log 10). \tag{15}$$

From (14) and (15), we find

$$k\log\alpha - \log 1.65 < 5.26\cdot 10^{12}\cdot(1+\log(k+15))\left(25.02 + 3[8.58\cdot 10^{13}\cdot(1+\log(k+15)) + \log(9.81)]\right).$$

It can be shown that $573 \le k < 2.59\cdot 10^{31}$. Now, we take

$$z_1 := -n\cdot\log 10 + \log\left(9\frac{t}{d_1}\right) + k\cdot\log\alpha \tag{16}$$

and

$$z_2 := \log 10\cdot n - k\cdot\log\alpha + \log\left((d_1 - d_2 10^{m-n})/(9t)\right). \tag{17}$$

We write

$$|x| = \left|e^{z_1} - 1\right| < \frac{9.81}{10^{n-m}} < 0.99$$

and

$$\left|x'\right| = \left|e^{z_2} - 1\right| < \frac{1.65}{\alpha^k} < 0.1$$

for $n - m \ge 1$ and $k \ge 573$ from (8) and (9). Let's choose $a := 0.99$ and $a' := 0.1$ to use Lemma 3. Thence, we obtain the inequalities

$$|z_1| = |\log(x+1)| < \frac{\log(100)}{(0.99)}\cdot\frac{9.81}{10^{n-m}} < 45.7\cdot 10^{m-n} \tag{18}$$

and

$$|z_2| = \left|\log(x'+1)\right| < \frac{\log(10/9)}{(1/10)}\cdot\frac{1.65}{\alpha^k} < 1.74\cdot\alpha^{-k}. \tag{19}$$

From (16), (17), (18), and (19), we get

$$
\begin{aligned}
&0 < |k \cdot \log \alpha - n \cdot \log 10 + \log(9t/d_1)| < 45.7 \cdot 10^{m-n}, \\
&0 < \left| \left(\frac{\log \alpha}{\log 10} \right) k - n + \left(\frac{\log(9t/d_1)}{\log 10} \right) \right| < 19.9 \cdot 10^{m-n},
\end{aligned} \tag{20}
$$

and

$$
\begin{aligned}
&0 < \left| n \cdot \log 10 - k \cdot \log \alpha + \log\left((d_1 - d_2 10^{m-n})/(9t) \right) \right| < 1.74 \cdot \alpha^{-k}, \\
&0 < \left| n \left(\frac{\log 10}{\log \alpha} \right) - k + \frac{\log\left((d_1 - d_2 10^{m-n})/(9t) \right)}{\log \alpha} \right| < 6.19 \cdot \alpha^{-k}.
\end{aligned} \tag{21}
$$

If we take $\gamma := \dfrac{\log \alpha}{\log 10} \notin \mathbb{Q}$, $\gamma' := \dfrac{\log 10}{\log \alpha} \notin \mathbb{Q}$ and $M := 2.59 \cdot 10^{31}$, it can be shown that the denominator of the 72nd convergent of γ, q_{72}, exceeds $6M$ and the denominator of the 136th convergent of γ' exceeds $6M$. Firstly, we apply Lemma 2 for $\gamma := \dfrac{\log \alpha}{\log 10}$ and take

$$\mu := \log(9t/d_1)/\log 10.$$

Since $1 \leq d_1 \leq 9$, it can be shown that the inequality

$$0.08 < \epsilon(\mu) := ||\mu q_{72}|| - M||\gamma q_{72}|| < 0.47.$$

In Lemma 2, suppose that $(A, B, w) := (19.9,\ 10,\ n-m)$. At that case, there is no solution to the inequality (20) if

$$\log(19.9 \cdot q_{72}/\epsilon)/\log 10 < 35.8 < n - m.$$

That's why

$$n - m \leq 35$$

and from (15), it follows that $k < 2.1 \cdot 10^{17}$. Similarly, we take $\gamma' := \dfrac{\log 10}{\log \alpha} \notin \mathbb{Q}$, $M := 2.1 \cdot 10^{17}$, and

$$\mu := \frac{\log\left((d_1 - d_2 10^{m-n})/(9t) \right)}{\log \alpha}.$$

For $1 \leq n - m \leq 36$ and $1 \leq d_1, d_2 \leq 9$, we have

$$0.00005 < \epsilon(\mu) = ||\mu q_{136}|| - M||\gamma q_{136}|| < 0.4999.$$

In Lemma 2, suppose that $(A, B, w) := (6.19, \alpha, k)$. At that case, there is no solution to the inequality (21) if

$$\log(A \cdot q_{136}/\epsilon)/\log B < 568.85 < k.$$

That's why $k \leq 568$. Since $k \geq 573$, this is impossible. □

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